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THE

TUTOR'S ASSISTANT;

BEING A

COMPENDIUM OF ARITHMETIC,

AND

A COMPLETE QUESTION-BOOK FOR THE USE OF SCHOOLS.

BY

FRANCIS WALKINGAME.

A New and Improved Edition,

TO WHICH IS ADDED,

AN APPENDIX ON THE DECIMAL COINAGE.

By J. R. YOUNG,

LATE PROFESSOR OF MATHEMATICS, BELFAST COLLEGE.

LONDON:
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1855.

181. c. 60.

PREFACE.

THE work of which a new and improved edition is here presented to the public, has for nearly a hundred years enjoyed an amount of favour from Schoolmasters and Teachers far surpassing that which any other book on ARITHMETIC has ever received.

It is of importance, therefore, in a performance of such universal acceptance, and exercising such extensive influence in the machinery of education, that sound principles should be inculcated throughout. If error be anywhere taught, it becomes, in so far as the error prevails, an instrument of mischief rather than of good.

In all the editions that have been consulted during the preparation of the present one, what is, perhaps, the most important rule in arithmetic—the Rule of Three—is based upon erroneous views of the doctrine of proportion. It is true that by one wrong step opposing and counteracting another, the influence of both is in the end neutralized, and the correct result arrived at; but nothing can justify a departure from scientific accurary in a book on arithmetic; and the method alluded to is opposed equally to the true principles of science and to the dictates of common sense: the blemish is removed in the present edition.

Besides this necessary modification in the mode of stating a question in the Rule of Three—and which is introduced throughout, wherever proportion is concerned,—a new rule has been inserted, for the more easy management of examples in the Double Rule of Three, or Compound Proportion; and many other improvements and simplifications will be found to have been supplied in different parts of the work, in Geometrical Progression, for instance, pp. 87-8, and in what concerns Interest and Annuities.

But as to plan, and arrangement, and practical illustration by examples, Walkingame has been substantially untouched. It is certainly overstepping his province for an editor to *replace* his author's work by his own, except in so *far as the* removal of errors and obscurities may justify

The purchasers of "Walkingame's Arithmetic" have surely a right to something more of the author than his title-page; and yet in some editions all resemblance to the original book, excepting the title-page, is obliterated.

The editions upon which the present is more especially based, are the fifteenth and twenty-first, both by Walkingame himself—the latter bears date 1785. This and all the intermediate editions are merely reprints of the fifteenth.

The book is more especially valuable for its copious variety of examples, and extensive range of subjects; and, in its present more modernized, improved, and corrected form, it is hoped that it may continue to sustain the character it has so long possessed, as a favourite school-book of practical arithmetic.

THE Publishers take this opportunity to announce that it is their intention to supply a simple and easy COURSE OF MATHEMATICS, adapted to School purposes, as well as for the instruction of those who may be preparing for the Universities, for the Naval or Military professions, or for Mechanical and Engineering pursuits.

The work will be comprised in three volumes, printed in the same type as that now before the reader; and as it is intended to render each volume complete in itself, they will be published, and may be purchased, separately.

The range of subjects will be pretty nearly co-extensive with that of Hutton's Course, as enlarged by Dr. Gregory—a work, the wide circulation of which, in numerous editions, largely contributed to awaken that taste for scientific pursuits, and to stimulate that spirit of inquiry, the practical results of which are developing themselves in our own day. The *treatment* of these subjects however, will, in many respects, be different: the latest improvements will be introduced, and the antiquated notation, in certain parts of Hutton, abandoned.

It is intended to publish the first volume, to contain the principles of ALGEBRA and TRIGONOMETRY, with their practical applications, early in January, 1855, and the second and third volumes at intervals of three months: the number of pages in each will be about 224, and the price as low as possible.

The Publishers conceive that there is need of a work of this kind: at all events they feel that they are not infringing upon pre-occupied ground. There exists no School Course of theoretical and practical Mathematics at present.

They have only to add, that the preparation of the Course is committed to Professor Young: those who are acquainted with his mathematical writings can form an opinion as to how the work will be executed.

Farringdon Street, September, 1854.

CONTENTS.

PART I.

ARITHMETIC IN WHOLE NUMBERS.

Introduction ..... *Page* 1
Numeration 1
Addition 3
Subtraction 4
Multiplication 5
Division 8
Tables of Money, Weights, Measures, &c. 10
Addition of Money, Weights, and Measures 17
Subtraction of Money, Weights, and Measures 21
Multiplication of several Denominations 25
Division of several Denominations 28
Bills of Parcels 29
Reduction 32
The Single Rule of Three Direct 41
The Rule of Three Inverse 44
The Double Rule of Three 46
Practice 49
Simple Interest 55
Commission 56
Purchasing of Stocks 56
Brokerage 57
Compound Interest 63
Discount 64
Equation of Payments 65
Barter 66
Profit and Loss 67
Fellowship without Time 68
Fellowship with Time 70
Alligation Medial 7

Alligation Alternate ..... *Page*
Alligation Partial .....
Alligation Total .....
Position, or the Rule of False .....
Single Position .....
Double Position .....
Exchange.....
Comparison of Weights and Measures .....
Conjoined Proportion .....
Arithmetical Progression .....
Geometrical Progression .....
Permutation .....

PART II.

VULGAR FRACTIONS.

Reduction .....
Addition .....
Subtraction.....
Multiplication .....
Division .....
The Single Rule of Three Direct.....
The Single Rule of Three Inverse .....
The Double Rule of Three .....

PART III.

DECIMAL FRACTIONS.

Numeration .....
Addition .....
Subtraction.....
Multiplication.....
Contracted Multiplication.....
Division .....
Contracted Division .....
Reduction .....
Decimal Tables of Coin, Weight, and Measure.....
The Rule of Three in Decimals .....
Extraction of the Square Root.....
Square root of a vulgar Fraction .....

Square root of a mixed Number ..... *Page* 112
Extraction of the Cube Root 115
Extraction of the Biquadrate Root 118
A General Rule for Extracting the Roots of all Powers 119
Simple Interest 120
Interest for Days 121
Annuities, Pensions, &c. in Arrears 124
Present Worth of Annuities 126
Annuities, &c. taken in Reversion 129
Rebate, or Discount (Simple Interest) 131
Equation of Payments 133
Compound Interest 134
Annuities, or Pensions, in Arrears 136
Present Worth of Annuities 138
Annuities, Leases, &c. taken in Reversion 140
Purchasing Freehold or Real Estates 141
Rebate, or Discount (Compound Interest) 143

PART IV.

DUODECIMALS.

Multiplication of Feet and Inches 146
Measuring by the Foot Square 147
Measuring by the Yard Square 148
Measuring by the square of 100 Feet 148
Measuring by the Rod 149

PART V.

QUESTIONS.

A Collection of Questions, set down promiscuously, for further exercise in the foregoing Rules 150
A general Table, for calculating Interest, Rents, Incomes, and Servants' Wages 157
A Table showing the number of days from any day in one month, to the same day in any other month, through the year 159

APPENDIX.

On the Decimal Coinage 161

EXPLANATION OF THE CHARACTERS MADE USE OF IN THIS COMPENDIUM.

= *Equal.*	The sign of Equality; as, 4 qrs. = 1 cwt., signifies that 4 qrs. are equal to 1 cwt.
− *Minus, or less.*	The sign of Subtraction; as, 8 − 2 = 6; that is, 8 diminished by 2 is equal to 6.
+ *Plus, or more.*	The sign of Addition; as, 4 + 4 = 8; that is, 4 added to 4 more is equal to 8.
× *Multiplied by.*	The sign of Multiplication; as, 4 × 6 = 24; that is, 4 multiplied by 6 is equal to 24.
÷ *Divided by.*	The sign of Division; as, 8 ÷ 2 = 4; that is, 8 divided by 2 is equal to 4.
$\frac{2357}{63}$	Numbers placed like a fraction likewise denote division; the upper number being the dividend, and the lower the divisor.
: :: :	The sign of Proportion; as, 2 : 4 :: 8 : 16; that is, as 2 is to 4, so is 8 to 16.
7 − 2 + 5 = 10	Shows that the difference between 2 and 7, added to 5, is equal to 10.
9 − (2 + 5) = 2	Signifies that the sum of 2 and 5, taken from 9, is equal to 2.
4^2, 2^3, *&c.*	A small figure placed at the upper corner of another figure, denotes a power of the latter, or the multiplication of it by itself; thus, 4^2, the second power or square of 4, is 4 × 4 = 16; also 2^3, the third power or cube of 2, is 2 × 2 × 2 = 8; and 2^4, the fourth power of 2, is 16.
$\surd$	Prefixed to any number, signifies that the square root of that number is to be understood.
$\sqrt[3]{}$	Signifies the Cube, or third root.
$\sqrt[4]{}$	Denotes the Biquadrate, or the fourth root, &c.

THE

TUTOR'S ASSISTANT,

BEING

A COMPENDIUM OF ARITHMETIC.

PART I.

ARITHMETIC OF WHOLE NUMBERS.

INTRODUCTION.

ARITHMETIC is the art or science of computing by *numbers*.

The theory of arithmetic considers the nature and properties of numbers, and demonstrates the reasons of the practical operations.

The practice shows the methods of working by numbers, in a manner the most useful and expeditious for business; it has five principal or fundamental *Rules* for its operations, namely, Numeration, Addition, Subtraction, Multiplication, and Division.

NOTE.—Numbers, as *one, two, three,* &c., may be expressed in *words*, as here shown; but in arithmetic, they are denoted by marks called *figures*, thus, 1, 2, 3, &c. There are only nine of these marks or figures, and a tenth mark for *nought* or *nothing*, namely 0; so that all the marks used in numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; each representing so many *units* or *ones*. How to express numbers, consisting of two or more figures, is shown in the Numeration Table on next page.

NUMERATION.

Numeration teaches the different values of figures, according to the places they occupy in a number, and explains how any *number* may be written down or expressed in *words*. The following table sufficiently shows how this is to be done; thus, the number 321 is three *hundreds*, two *tens*, and one *unit*, that is, three hundred and twenty-one; the number 4321 is four thousand three hundred and twenty-one; the number *54321 is fifty-four* thousand three hundred and twenty-one, *and so on.*

THE NUMERATION TABLE.

4	3	2	1	9	8	7	6	5	4	3	2	1
billions.	hundreds of thousands of millions.	tens of thousands of millions.	thousands of millions.	hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.

To read a large number, the easiest way is to separate the figures into *periods* of three figures each, beginning at the *right hand*, observing that, by the above table, the leading figure of each period is always *hundreds;* thus, the leading figure 3 of the first period is 3 *hundreds;* the leading figure 6 of the second period is 6 *hundreds* of thousands; the leading figure 9 of the third period is 9 *hundreds* of millions, and so on. To separate the figures of a large number into periods, a comma is generally used; thus, the number 4682937, with the periods marked, is 4,682,937, and as the leading figure 6 of the second period is six hundred thousand, the preceding figure 4 must be 4 millions; so that the number is read four millions six hundred and eighty-two thousand nine hundred and thirty-seven.

Write down in proper Figures the following Numbers.

Twenty-three.
Two hundred and fifty-four.
Three thousand two hundred and four.
Twenty-five thousand eight hundred and fifty-six.
One hundred and thirty-two thousand two hundred and forty-five.
Four millions nine hundred and forty-one thousand four hundred.
Twenty-seven millions one hundred and fifty-seven thousand eight hundred and thirty-two.
Seven hundred and twenty-two millions two hundred and thirty-one thousand five hundred and four.
Six hundred and two millions two hundred and ten thousand five hundred.

Write down in Words at Length the following Numbers.

35	2017	519007	5207054	65700047
59	5201	750058	2071909	900061057
172	20760	5900030	70054008	201900790

Notation by ROMAN *Letters.*

I. One.
II. Two.
III. Three.
IV. Four.
V. Five.
VI. Six.
VII. Seven.
VIII. Eight.
IX. Nine.
X. Ten.
XI. Eleven.
XII. Twelve.
XIII. Thirteen.
XIV. Fourteen.
XV. Fifteen.
XVI. Sixteen.
XVII. Seventeen.
XVIII. Eighteen.
XIX. Nineteen.
XX. Twenty.
XXX. Thirty.
XL. Forty.
L. Fifty.
LX. Sixty.
LXX. Seventy.
LXXX. Eighty.
XC. Ninety.
C. Hundred.
CC. Two Hundred.
CCC. Three Hundred.
CCCC. Four Hundred.
D. Five Hundred.
DC. Six Hundred.
DCC. Seven Hundred.
DCCC. Eight Hundred.
DCCCC. Nine Hundred.
M. One Thousand.
MDCCCLV. One Thousand Eight Hundred and Fifty-Five.

ADDITION.

ADDITION teaches how to add two or more numbers together, so as to make one whole or total sum.

RULE. There must be due regard had to placing the figures one under the other, that is, units under units, tens under tens, &c., then beginning with the first row, the row of units, add them up to the top, when done, set down the units, and carry the tens to the next row, and so on, continuing to the last row, at which set down the total amount.

PROOF. Begin at the top of the sum, and reckon the figures downwards, the same as you added them up, and, if the result is the same as at first, the sum is supposed to be right.

ADDITION AND SUBTRACTION TABLE.

	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

This table gives the sum of any two numbers between 1 and 10. For example, if the two numbers are 8 and 7, you point to the 8 in the fir

column on the left, and then pass your finger along the row of figures to the right of the 8, till you come to that number which is directly under the 7 in the top row; this number is seen to be 15, which is the sum required; or you may point to the 7 in the first column, and take that number to the right of this 7 which is directly under the 8 in the top row, which number, as before, is 15. The table helps a beginner in this way in his first attempts at addition.

Examples.

Ex. (1)	(2)	(3)	(4)	(5)	(6)
2	27	276	1234	7524	27104
5	35	110	7098	3750	32547
7	47	473	3314	9147	10758
9	35	354	6732	3215	62590
2	41	271	2546	4725	75408
5	36	352	3709	2147	27973
4	39	471	4152	3254	85421
3	57	310	3705	2716	12706
7	14	437	1076	1047	10471
44	331				

SUBTRACTION.

SUBTRACTION teaches how to take a less number from a greater, and thus to find the remainder or difference between the two.

RULE. Write the less number under the greater, taking care that units may be placed under units, tens under tens, and so on, as in addition.

Then, beginning at the units' place, subtract each figure in the lower row from the figure above it, observing that if the lower figure be greater than the upper, to add 10 to that upper figure, and then subtract, putting down the remainder and carrying 1 to the next lower figure. The next *lower* figure is thus to be increased by 1 only when 10 has been added to the preceding *upper* figure.

To prove Subtraction. Add the remainder to the lower or less number; the sum will be the upper or greater number, if the work is correct.

Examples.

	(1)	(2)	(3)	(4)	(5)	(6)
From	271	3754	42087	452705	271508	3750214
Take	154	2725	35096	327616	152471	2150873
Rem.	117					
Proof	271					

MULTIPLICATION.

MULTIPLICATION teaches how to increase the greater of two given numbers as many times as there are units in the less; and compendiously performs the office of many additions.

The operation consists of three principal members, viz. :—

1. The *Multiplicand*, or number to be multiplied.
2. The *Multiplier*, or number by which you multiply.
3. The *Product*, or number produced by multiplying.

There are two cases to be considered: First, when the multiplier is not greater than 12, and, Second, when the multiplier *is* greater than 12. The rule for the first case is this, viz. :—

RULE 1. Place the multiplier under the multiplicand, units under units, and by it multiply the first or units' figure of the multiplicand. Set down the units' figure of the product, and carry the tens' figure in mind till you have multiplied the next figure of the multiplicand by your multiplier, when the tens' figure you kept in mind must be added to the product, and the units set down and the tens carried as before, and so on, till the whole row is multiplied.

By the following table, the product of any two numbers, each of which is not greater than 12, may be found; thus, if the product of 7 and 9 is sought, point to the 7 in the first column on the left, then pass your finger along the row of figures on the right of this 7, stopping at the number in that row which is directly under the 9 in the top row: this number is seen to be 63. Or point to the 9 in the left-hand column, and proceed along the row on the right of it till you reach the number under the 7 in the top row, which, as before, is 63, the product required. This table should be learnt, till you are able to repeat it without the book.

MULTIPLICATION TABLE.

	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Examples.

Multiplicand	25104736	52471021	7925437521
Multiplier	2	3	4
Product	50209472		

27104107	231047	7092516	3725104
5	6	7	8
4215406	2701047	31040171	35210472
9	10	11	12

For the second case, when the multiplier is a number greater than 12, the rule is as follows :—

RULE 2. Place the multiplier under the multiplicand, units under units, as before, and multiply by each figure of the multiplier separately, as in the former case, but observe to put the first figure of every product under the figure you multiply by ; you will thus get as many rows of figures as there are figures in the multiplier (not counting the ciphers). Add these rows up, as in addition ; the sum will be the total product.

To prove Multiplication. The usual way of proving multiplication is by "the method of casting out the nines," as it is called ; this method is as follows :—Add up the figures of the multiplicand, but as you proceed always reject 9 as soon as your sum amounts to that or to a greater number, and carry on the addition with only what remains. Put the entire sum, after all these rejections, on one side of a cross, as in example 1, following.

Add up the figures of the multiplier in like manner, putting the sum, after the nines are cast out, on the opposite side of the cross.

From the product of the numbers on each side of the cross, or, which is the same thing, from the sum of the figures in that product, reject the nines, and place the result above the cross. Proceed in the same way with the figures of the product, placing the result below the cross ; then, if this last result be the same as that before placed above the cross, the product may be presumed to be right ; but if the two results differ, the work is certainly wrong.

Examples.

0	(1) 271041	(2) 32104	(3) 2710432	(4) 27501976
6 × 0	27	25	375	271
0	1897287	802600	1016412000	7453035496
	542082			
	7318107			

When ciphers are placed between the significant figures in the multiplier, they may be omitted ; but great care must be taken to put *the first figure on the right* in each row, under the figure you multiply *by.*

(5) 246385	(6) 571204	(7) 7104325	(8) 5271094
3062	27009	57020	590030
492770	15427648836	405058611500	3110103592820
1478310			
739155			
754430870			

When there are ciphers at the end of multiplicand or multiplier, they may be omitted, by only multiplying by the rest of the figures, and setting down on the right hand of the total product as many ciphers as were omitted, as in the following examples.

(9) 27100	(10) 379500	(11) 265000	(12) 574000
52600	274000	7200	630
1425460000	103983000000	1908000000	361620000

When the multiplier is a *composite number*, that is, if any two figures, being multiplied together, will make that number, then multiply by one of those figures, and that product by the other will give the answer.

(13) 3628 by 24, or 4 times 6	(14) 771039 by 35	(15) 921563 by 32
4	26986365	29490016
14512		
6	(16) 715241 by 56	
87072	40053496	

NOTE.—Whenever the multiplier is a number less than 20, the product may always be written down at once, and two rows of figures to be added up avoided; thus, for all multipliers between 10 and 20, proceed as follows: multiply by the unit's figure of the multiplier, adding to the product the next back figure of the multiplicand; thus, in the first of the following examples, you would say, 3 times 2 are 6; 3 times 9 are 27, which, with the back figure 2, make 29, 9 and carry 2; 3 times 5 are 15 and 2 carried are 17, which, with the back figure 9, make 26, 6 and carry 2; and so on.

5710592	5107252	7653210	92057165
13	14	15	16
74237696			
6251721	9215324	2571341	3592104
17	18	19	19

Additional Examples for practice.

NOTE.—In these examples, the sign × means "multiplied by :" it is called the sign of multiplication ; and the sign = means "equal to :" it is called the sign of equality.—(See explanation of signs at the beginning.)

(1) $296 \times 105 = 31080$
(2) $345 \times 345 = 119025$
(3) $2071 \times 227 = 470117$
(4) $9250 \times 570 = 5272500$
(5) $488601 \times 699 = 341532099$
(6) $25103 \times 2518 = 63209354$
(7) $29 \times 31 \times 27 = 24273$
(8) $365 \times 365 \times 365 = 48627125$

DIVISION.

DIVISION teaches to find how often one number is contained in another : or to divide any number into as many equal parts as you please.

In division, there are four principal things to be attended to.

1. The *Dividend,* or number to be divided.
2. The *Divisor,* or number to divide with.
3. The *Quotient,* or number that shows how often the divisor is contained in the dividend, and
4. The *Remainder,* or what is left over when the work is finished. There is always a remainder whenever the dividend does not contain the divisor a certain number of times exactly. There are two cases to be considered, First, when the divisor is not greater than 12 ; Second, when the divisor exceeds 12.

CASE 1. *When the Divisor is not greater than* 12.

RULE. Find how often the divisor is contained in the first figure of the dividend, or, if this first figure be not so great as the divisor, find how often it is contained in the number formed by the first two or the first three figures of the dividend ; set the quotient down under those figures, and carry the overplus (if any) to the next figure of the dividend, as so many *tens ;* then find how many times the divisor is contained in this new number, set down the quotient underneath, and continue the operation till you have gone through the row of figures in the dividend ; what is left at the end is the remainder.

For the Proof. Multiply the divisor and entire quotient together, adding in the remainder (if any), and the product will be the same as the dividend, if the work is right.

	(1) Divisor 2)725107	(2) 3)7210472	(3) 4)7210416
Quotient	362553..1 rem.	2403490..2	1802604
	2	3	4
Proof	725107	7210472	7210416

(4) 2)538941

(5) 3)1368400

(6) 4)3700623

(7) 5)7203287

(8) 6)5231047

(9) 7)2532701

(10) 11)2750012

(11) 12)72100732

(12) 12)101236593

CASE 2. *When the Divisor is greater than* 12.

RULE. As before, find how many times the divisor is contained in the number formed by the leading figures of the dividend, taking no more of these figures than are sufficient to form a number as great, at least, as the divisor.

The first quotient figure being thus obtained, multiply the divisor by it, subtract the product from the dividend, and to the remainder annex the following figure of the dividend; and proceed with the operation till all the figures are brought down.

The method of proof is the same as in the former case.

Examples.

Divisor. Dividend. Quotient.

(1) 29)4172377(143875

29

127

116

112

87

253

232

217

203

147

145

Rem. 2

1294875

287750

2 rem.

4172377 Proof.

(2) 37)7210473(194877

(3) 473)2104721(4449

(4) 275)3720147(13527

(5) 3701)72109521(19483

(6) 3576)72104725(20163

(7) 2510)63210476(25183

(8) 25204)321047217(12737

(9) 31709)521047321(16432

(10) 2701234)7210472532(2669

(11) 210472)352107193214(1672940

(12) 3721071)21071921473(5662

When there are ciphers at the end of the divisor, they may be cut off, and *as many places* from off the dividend, but these last must be *annexed to the remainder.*

Examples.

Divisor.	Dividend.	Quotient.
(1) 237	00)56937	28(248
	474	
	1153	
	948	
	2057	
	1896	
	16128 rem.	

Divisor.	Dividend.	Quotient.
(2) 349	000)178649	251(511
	1745	
	414	
	349	
	659	
	349	
	310251 rem.	

(3) 271|00)254782|21(939

(4) 373|000)752473|719(2006

(5) 5721|00)7253472|16(1267

(6) 215|000)6325104|997(29419

When the divisor is a composite number (*i. e.* if any two figures, being multiplied together, will make that number), then, by dividing the dividend by one of those figures, and that quotient by the other, it will give the quotient required; but as it sometimes happens that there is a remainder to each of the quotients, and neither of them the true one, it may be found by this

RULE. Multiply the first divisor into the second remainder, to that product add the first remainder, which will give the true one.

Thus, to divide 5603782 by 24, since 24 is = 4 times 6, you may divide first by one of these numbers, say 4: the quotient is 1400945 with 2 for remainder; then dividing by the other number 6, the quotient is 233490, with 5 for remainder; thus 5, multiplied by the first divisor 4 gives 20, and the first remainder 2 added to this gives 22 for the true remainder.

(1) Divide 3210473 by 27

118906..11 rem.

(2) 7210473 by 35

206313..18 rem.

(3) 6251043 by 42

148834..15 rem.

(4) 5761034 by 54

106685..44 rem.

TABLES OF MONEY, WEIGHTS, MEASURES, &c.

MONEY accounts are kept in pounds, shillings, pence, and farthings. The mark for *pounds* is £ or *l*, thus, £4 or 4*l*. means 4 pounds; the mark for *shillings* is *s*, thus 7*s*. means 7 shillings; and the mark for *pence is d*, thus 8*d*. means 8 pence; also ¼*d*. means one-fourth of a *penny, it is the mark for one farthing;* likewise ½*d*., or the half of one

penny, is the mark for a *halfpenny*, or 2 farthings; and ¾*d.*, in like manner, denotes 3 *farthings*.

MONEY.

Farthings.
4 = 1 Penny.
48 = 12 = 1 Shilling.
960 = 240 = 20 = 1 Pound.

PENCE TABLE.

	s.	d.		s.	d.		s.	d.
12 Pence	= 1	0	60 Pence	= 5	0	108 Pence	= 9	0
20 ,,	= 1	8	70 ,,	= 5	10	110 ,,	= 9	2
24 ,,	= 2	0	72 ,,	= 6	0	120 ,,	= 10	0
30 ,,	= 2	6	80 ,,	= 6	8	130 ,,	= 10	10
36 ,,	= 3	0	84 ,,	= 7	0	132 ,,	= 11	0
40 ,,	= 3	4	90 ,,	= 7	6	140 ,,	= 11	8
48 ,,	= 4	0	96 ,,	= 8	0	144 ,,	= 12	0
50 ,,	= 4	2	100 ,,	= 8	4			

The coins in present use in the United Kingdom are the following:

Copper Coins.

Penny, Half-penny, Farthing.

Gold Coins.

Sovereign, in value, 20*s.*, in weight, 5 dwts. $3\frac{171}{223}$ grs., troy weight.
Half-sovereign ,, 10*s.*, ,, 2 ,, $13\frac{212}{223}$,, ,,

Silver Coins.

Crown,	in value, 5*s.*,	in weight,	18 dwts.	$4\frac{4}{11}$ grs.
Half-crown	,, 2*s.* 6*d.*	,,	9 ,,	$2\frac{2}{11}$,,
Florin	,, 2*s.* 0*d.*	,,	7 ,,	$6\frac{6}{11}$,,
Shilling	,, 12*d.*	,,	3 ,,	$15\frac{3}{11}$,,
Sixpence	,, 6*d.*	,,	1 ,,	$19\frac{7}{11}$,,
4-penny piece	4*d.*	,,	1 ,,	$5\frac{1}{11}$,,
3-penny piece	3*d.*	,,	0 ,,	$21\frac{8}{11}$,,

NOTE.—Gold and silver coins are not entirely pure gold and silver; the former have an *alloy* of 2 parts of copper to 22 parts of pure gold; and the latter have an alloy of 18 dwt. of copper to 11 oz. 2 dwt. of pure silver. These mixtures are called respectively *Standard* gold and *Standard* silver; and the Mint standard gold is said to be 22 *carats* fine, while pure gold is 24 carats fine; a carat simply meaning a twenty-fourth part of the whole. In consideration of the loss of weight by wear, a sovereign is allowed to pass for its full value provided it weigh 5 dwt. 2¾ gr.

TROY WEIGHT.

			Marked.
24 grains.....	make	1 pennyweight	(gr.) (dwt.)
20 pennyweights	,,	1 ounce	(oz.)
12 ounces	,,	1 pound	(lb.)

Grains.
24 = 1 Pennyweight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

By this weight are weighed Gold, Silver, Jewels, Electuaries, and all liquids.

25 lb. is a Quarter of a cwt., 100 lb. = 1 cwt., and
20 cwt. = 1 Ton of Gold or Silver.

NOTE.—As observed in the preceding Note, the word *carat*, when used in reference to the purity of the precious metals, means merely the twenty-fourth part of the entire mass; but when the same term is employed in reference to the weight of diamonds it denotes 3½ grains. A diamond of the first quality (first *water*, as it is called), when properly cut, and without flaws, is worth £8, if it weigh 1 carat; it is worth £32 if it weigh 2 carats; £72 if it weigh 3 carats; £128 if it weigh 4 carats; and so on. The dust or refuse, made in cutting diamonds, is worth about £50 an ounce.

AVOIRDUPOIS WEIGHT.

			Marked.
16 drams	make	1 ounce	(dr.) (oz.)
16 ounces	,,	1 pound	(lb.)
28 pounds	,,	1 quarter	(qrs.)
4 quarters or 112 lb.	,,	1 hundred weight	(cwt.)
20 hundred weight	,,	1 ton	(ton.)

Drams.
16 = 1 Ounce.
256 = 16 = 1 Pound.
7168 = 448 = 28 = 1 Quarter.
28672 = 1792 = 112 = 4 = 1 Hundred weight.
573440 = 35840 = 2240 = 80 = 20 = 1 Ton.

There are several other denominations in this weight, that are used in some particular goods: viz.

	lb.		lb.
A Firkin of Butter	= 56	A Stone of Iron Shot, or Horseman's weight	= 14
,, Soap	= 94	,, Butcher's Meat	= 8
A Barrel of Anchovies	= 30	A Truss of Straw	= 36
,, Soap	= 256	,, New Hay	= 60
,, Raisins	= 112	,, Old Hay	= 56
A Puncheon of Prunes	= 1120	36 Trusses a Load.	
A Fother of Lead = 19 cwt. 2 qrs.			

CHEESE AND BUTTER.

A Clove, or Half Stone = 8 lb.
A Wey in Suffolk = 32 Cloves, or 256 lb.
A Wey in Essex = 42 Cloves, or 336 lb.

WOOL.

	lb.		lb.
A Clove	= 7	A Wey = 6 Tod 1 Stone, or	182
A Stone	= 14	A Sack = 2 Weys, or	364
A Tod	= 28	A Last = 12 Sacks, or	4368

By Avoirdupois weight are weighed things of a coarse nature, as all Grocery and Chandlery Wares, Bread, and all Metals but Silver and Gold.

NOTE.—1 Pound Avoirdupois is equal to 14 oz. 11 dwts. 15½ grs. Troy.

APOTHECARIES WEIGHT.

			Marked.
20 Grains	make	1 Scruple	(℈)
3 Scruples	,,	1 Dram	(ʒ)
8 Drams	,,	1 Ounce	(℥)
12 Ounces	,,	1 Pound	(lb.)

Grains.

20 = 1 Scruple.
60 = 3 = 1 Dram.
480 = 24 = 8 = 1 Ounce.
5760 = 288 = 96 = 12 = 1 Pound.

NOTE.—Apothecaries mix their medicines by this rule, but buy and sell their commodities by Avoirdupois weight.

The Apothecaries' pound and ounce, and the pound and ounce Troy, are the same, only differently divided and subdivided.

CLOTH MEASURE.

			Marked.
2¼ Inches	make	1 Nail	(n.)
4 Nails	,,	1 Quarter of a yard	(qrs.)
3 Quarters	,,	1 Flemish ell	(F.E.)
4 Quarters	,,	1 Yard	(yd.)
5 Quarters	,,	1 English ell	(E.E.)
6 Quarters	,,	1 French ell	(Fr.E.)

Inches.

2¼ = 1 Nail.
9 = 4 = 1 Quarter.
36 = 16 = 4 = 1 Yard.
27 = 12 = 3 = 1 Flemish ell.
45 = 20 = 5 = 1 English ell.
54 = 24 = 6 = 1 French ell.

LONG MEASURE.

				Marked.
3	Barleycorns	make	1 Inch	(bar.) (in.)
12	Inches	„	1 Foot	(ft.)
3	Feet	„	1 Yard	(yd.)
6	Feet	„	1 Fathom	(fath.)
5½	Yards	„	1 Rod, Pole, or Perch,	(rod, p.)
40	Poles	„	1 Furlong	(furl.)
8	Furlongs	„	1 Mile	(mi.)
3	Miles	„	1 League	(lea.)
60	Miles	„	1 Degree	(deg.)

Barleycorns.

3 = 1 Inch.
36 = 12 = 1 Foot.
108 = 36 = 3 = 1 Yard.
594 = 198 = 16½ = 5½ = 1 Pole.
23760 = 7920 = 660 = 220 = 40 = 1 Furlong.
190080 = 63360 = 5280 = 1760 = 320 = 8 = 1 Mile.

NOTE.—The great circle round the earth, called the *equator*, as also every *meridian*, is conceived to be divided into 360 equal parts or *degrees*, and each of these parts into 60 equal portions, called *minutes*. The length of each minute of the meridian is a nautical or geographical *mile*, which exceeds a land mile or statute mile by about $\frac{1}{7}$ part of the latter. A degree of the equator is 60 nautical miles, or about $69\frac{1}{12}$ land miles. But as the earth is somewhat flattened towards the poles, the degrees in high latitudes become longer; it is usual to take 69½ miles for a degree, which, however, is about $\frac{1}{4}$ of a mile more than the polar degree.

WINE MEASURE.

				Marked
4	Gills	make	1 Pint	(pt.)
2	Pints	„	1 Quart	(qt.)
4	Quarts	„	1 Gallon	(gal.)
10	Gallons	„	1 Anker of Brandy	(ank.)
18	Gallons	„	1 Runlet	(run.)
31½	Gallons	„	Half a Hogshead	(½hhd.)
42	Gallons	„	1 Tierce	(tierce.)
63	Gallons	„	1 Hogshead	(hhd.)
2	Hogsheads (126 gal.)	„	1 Pipe or Butt	(P. or butt.)
2	Pipes, or 4 Hogsheads	„	1 Tun	(tun.)

NOTE.—The number of gallons in a pipe of wine differs according to the kind of wine; a pipe of claret is 126 gallons, as above, but a pipe of port is 138 gallons, of sherry 130, of Madeira 110, of Bucellas 140, &c. In the questions in this work, the claret pipe is to be understood; that is to say, without specifying the kind of wine, a pipe will be always taken to mean 126 gallons.

ALE AND BEER MEASURE.

				Marked
2 Pints	make	1 Quart		(pts.) (qts.)
4 Quarts	„	1 Gallon		(gal.)
9 Gallons	„	1 Firkin		(fir.)
2 Firkins	„	1 Kilderkin		(kil.)
4 Firkins, or 2 Kilderkins	„	1 Barrel		(bar.)
1½ Barrel, or 54 Gallons	„	1 Hogshead		(hhd.)
2 Barrels	„	1 Puncheon		(pun.)
3 Barrels, or 2 Hogsheads	„	1 Butt		(butt.)

NOTE.—A Barrel of Salmon or Eels = 42 gallons.
A Barrel of Herrings = 32 gallons.
A Keg of Sturgeon = 4 or 5 gallons.
A Firkin of Soap = 8 gallons.

Till the year 1803, the firkin of ale contained only 8 gallons, but by an Act of Parliament then passed, the firkin was fixed to contain 9 gallons, whether of ale or beer. The gallon was also a different measure for ale or beer from that for wine till 1826, but from the 1st of January in that year, it was commanded by Act of Parliament that the gallon should be of the same capacity throughout the United Kingdom for all liquids, without exception. It is called the *imperial gallon*, and holds just 10 pounds avoirdupois of distilled water, and the solid measure of this quantity is 277 cubic inches and 274 thousandths of an inch, which is 277¼ cubic inches, very nearly. For all practical purposes, the imperial gallon may be considered as measuring 277¼ cubic inches, exactly, since the exact capacity even of a butt, or 108 gallons, is not so much as half a cubic inch below the capacity, as determined by taking the above as the measure of a gallon; the error in a butt thus amounts to less than the 69th part of a pint.

Cub. In.	Pints.	Quarts.						
277¼ =	8 =	4 =	1 Gallon.					
2495¼ =	72 =	36 =	9 =	1 Firkin.				
4991 =	144 =	72 =	18 =	2 = 1	Kilderkin.			
9982 =	288 =	144 =	36 =	4 = 2 = 1	Barrel.			
14973 =	432 =	216 =	54 =	6 = 3 = 1½ = 1	Hogshead.			
19964 =	576 =	288 =	72 =	8 = 4 = 2 = 1	Puncheon.			
29946 =	864 =	432 =	108 =	12 = 6 = 3 = 2 = 1	Butt.			

DRY MEASURE.

			Marked
2 Pints	make	1 Quart	(pts.) (qts.)
4 Quarts	„	1 Gallon	(gal.)
2 Gallons	„	1 Peck	(pk.)
2 Pecks	„	1 Strike	(strike.)
2 Strikes, or 4 Pecks	„	1 Bushel	(bush.)
4 Bushels	„	1 Coomb	(coomb.)
2 Coombs, or 8 Bushels	„	1 Quarter	(qr.)
4 Quarters	„	1 Chaldron	(chald.)
5 Quarters	„	1 Wey	(wey.)
2 Weys	„	1 Last	(last.)

Cub. In.							
277¼ =	1 Gallon.						
554½ =	2 =	1 Peck.					
1109 =	4 =	2 =	1 Strike.				
2218 =	8 =	4 =	2 =	1 Bushel.			
8873 =	32 =	16 =	8 =	4 =	1 Coomb.		
17745½ =	64 =	32 =	16 =	8 =	2 =	1 Quarter.	
70982 =	256 =	128 =	64 =	32 =	8 =	4 =	1 Chaldron.
88727½ =	320 =	160 =	80 =	40 =	10 =	5 =	1 Wey.
177455 =	640 =	320 =	160 =	80 =	20 =	10 =	2 = 1 Last.

A Score of Coals.....	=	21 Chaldrons.
A Sack of Coals	=	3 Bushels.
A Chaldron of Coals	=	12 Sacks, or 36 Bushels.
A Load of Corn	=	5 Bushels.
A Cartload of Corn.....	=	40 Bushels.

By an Act of Parliament, passed in 1831, coals, within a distance of 25 miles from the General Post-office, London, are to be sold by *weight*, and not by heaped measure, as was previously done. The legal weight of 1 sack of coals is 2 cwt., or 224 lbs.; so that 10 sacks make 1 ton. But in many country places, coals are still sold by heaped measure, and the *chaldron*, now abolished in London, is sometimes used elsewhere.

TIME.

			Marked
60 Seconds	make	1 Minute	(sec.) (m.)
60 Minutes.....	,,	1 Hour	(h.)
24 Hours	,,	1 Day	(d.)
7 Days	,,	1 Week	(w.)
4 Weeks	,,	1 Month	(mo.)
13 Months 1 Day 6 Hours	,,	1 Julian Year	(yr.)

Seconds.									
60 =	1 Minute.								
3600 =	60 =	1 Hour.							
86400 =	1440 =	24 =	1 Day.						
604800 =	10080 =	168 =	7 =	1 Week.					
2419200 =	40320 =	672 =	28 =	4 =	1 Month.				
			d.	h.		w.	d.	h.	
31557600 =	525960 =	8766 =	365	6 =	52	1	6 =	1 Julian year.	
			d.	h.	m.	sec.			
31556937 =	525948 =	8765 =	365	5	48	57 =	1 Solar year.		

To know the days in each month observe—

Thirty days hath September,
April, June, and November,
February hath *twenty-eight* alone,
And all the rest have *thirty-one*.
Except in Leap-Year, and then's the time,
February's days are *twenty and nine*.

SQUARE MEASURE.

144	Inches	make	1 Foot.
9	Feet	,,	1 Yard.
100	Feet	,,	1 Square of flooring.
272¼	Feet	,,	1 Rod.
40	Rods	,,	1 Rood.
4	Roods, or 160 Rods, or 4840 Yards	,,	1 Acre of Land.
640	Acres	,,	1 Square Mile.
30	Acres	,,	1 Yard of Land.
100	Acres	,,	1 Hide of Land.

By this measure are measured all things that have length and breadth: such as land, painting, plastering, flooring, thatching, plumbing, glazing, &c.

Inches.				
144 =	1 Foot.			
1296 =	9 =	1 Yard.		
39204 =	272¼ =	30¼ =	1 Pole.	
568160 =	10890 =	1210 =	40 =	1 Rood.
6272640 =	43560 =	4840 =	160 = 4 =	1 Acre.

SOLID OR CUBIC MEASURE.

1728	Inches	make	1 Solid Foot.
27	Feet	,,	1 Yard or Load of Earth.
40 Feet of Round Timber Or, 50 Feet of Hewn Timber		,,	1 Ton or Load.

108 solid feet, *i.e.* 12 feet in length, 3 feet in breadth, and 3 feet deep, or, commonly, 14 feet long, 3 feet 1 inch broad, and 3 feet 1 inch deep, is a stack of wood.

128 solid feet, *i.e.* 8 feet long, 4 feet broad, and 4 feet deep, is a cord of wood.

A ton of shipping is 42 solid feet. A solid yard of earth is 1 load. A cubic foot of water weighs 1000 ounces avoirdupois, or 911 oz. 9 dwt. 4 gr. troy

By this measure are measured all things that have length, breadth, and thickness or depth.

ADDITION OF MONEY, WEIGHTS, AND MEASURES.

RULE. Add the first row or denomination together, as in integers; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the row added, and carry *the quotient to* the next superior denomination, continuing *the same to the last,* which add as in simple addition.

MONEY.

(1)			(2)			(3)			(4)		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
2	13	5½	27	7	2	35	17	3	75	3	7
7	9	4¼	34	14	7½	59	14	7½	54	17	1½
5	15	4½	57	19	2¼	97	13	5¼	91	15	4¼
9	17	6¼	91	16	1	37	16	8¼	35	16	5¾
7	16	3	75	18	7¾	97	15	7	29	19	7¼
5	14	7¾	97	13	5	59	16	5½	91	17	3¼
39	6	7¼									

(5)			(6)			(7)			(8)		
257	1	5½	525	2	4¼	21	14	7¼	73	2	1½
734	3	7¾	179	3	5	75	16	0	25	12	7
595	5	3	250	4	7¼	79	2	4¼	96	13	5½
159	14	7½	975	3	5¼	57	16	5½	76	17	3¼
207	5	4	254	5	7	26	13	8¾	97	14	1
798	16	7¼	379	4	5¾	54	2	7	54	11	7¾

(9)			(10)			(11)			(12)		
127	4	7¼	261	17	1¼	31	1	1½	27	13	5
525	3	5	379	13	5	75	13	1	16	7	9¼
271	0	5	257	16	7¾	39	19	7½	9	15	3
524	9	1	184	13	5	97	17	3¼	15	2	7¼
379	4	3½	725	2	3¼	36	13	5	37	19	1
215	5	8¾	359	6	5	24	16	3¼	56	19	1¾

TROY WEIGHT.

(1)			(2)			(3)			(4)		
oz.	dwt.	gr.	oz.	dwt.	gr.	lb.	oz.	dwt.	lb.	oz.	dwt.
7	15	21	5	11	4	7	1	2	5	2	15
3	17	6	7	19	21	3	2	17	3	1	17
2	5	15	3	15	14	5	1	15	3	7	15
3	16	19	7	19	22	7	10	11	9	1	13
9	18	23	9	18	15	2	7	13	3	9	7
7	15	14	8	13	12	3	11	16	5	2	15

AVOIRDUPOIS WEIGHT.

(1)			(2)			(3)			(4)		
lb.	oz.	dr.	lb.	oz.	dr.	cwt.	qrs.	lb.	ton.	cwt.	qrs.
152	15	15	17	12	3	25	1	17	7	17	2
272	14	10	23	15	6	72	3	26	5	5	3
303	15	11	31	11	14	54	1	16	2	4	1
255	10	4	97	0	9	24	1	16	3	18	2
973	6	2	48	7	15	17	0	19	7	9	3
635	13	13	79	10	6	55	2	16	8	5	1

APOTHECARIES WEIGHT.

(1)			(2)				(3)				(4)			
℥	℈	gr.	℔	℥	℈	gr.	lb.	℔	℥	℈	lb.	℔	℥	℈
7	1	17	9	2	0	17	7	10	7	1	7	2	1	0
3	0	18	3	5	2	19	9	5	2	2	3	1	7	1
6	2	16	9	2	1	14	7	11	1	2	9	10	2	0
5	1	15	3	5	0	18	9	5	6	1	7	5	7	1
7	0	18	7	7	2	15	7	10	5	2	3	9	5	2
3	1	9	3	3	0	0	9	0	7	0	7	1	4	1

CLOTH MEASURE.

(1)			(2)			(3)			(4)		
F.E.	qrs.	n.	yds.	qrs.	n.	yds.	qrs.	n.	E.E.	qrs.	n.
27	2	1	35	3	3	73	3	2	72	2	1
15	1	3	70	2	2	97	1	3	52	1	2
37	0	2	95	3	0	54	0	2	79	0	1
52	1	3	76	1	3	76	2	0	56	2	0
76	2	1	26	0	1	59	1	3	79	3	1
97	1	3	79	2	1	76	2	2	54	2	1

LONG MEASURE.

(1)			(2)			(3)			(4)		
feet.	in.	bar.	yds.	feet.	in.	m.	fur.	p.	lea.	m.	fur.
27	9	2	25	1	9	35	7	3	72	2	1
35	10	1	71	0	3	27	5	27	27	1	7
17	2	0	52	2	3	52	0	35	35	2	5
35	11	1	97	0	10	97	1	17	79	0	6
97	2	2	54	2	7	56	7	18	51	1	6
54	8	1	37	1	4	91	5	27	72	0	5

LAND MEASURE.

(1)			(2)			(3)			(4)		
a.	r.	p.	a.	r.	p.	a.	r.	p.	a.	r.	p.
75	3	27	27	1	35	26	1	31	32	1	14
36	2	15	29	2	19	19	2	17	27	0	19
97	1	16	37	1	15	55	3	14	31	2	15
35	2	15	95	2	27	79	1	21	19	1	18
27	1	14	62	0	13	95	2	14	39	2	37

WINE MEASURE.

(1)			(2)			(3)			(4)		
run.	gal.	qts.	tier.	gal.	qts.	hhds.	gal.	qts.	tun.	hhds.	gal.
27	17	2	25	36	2	31	57	1	14	3	27
35	15	3	75	41	2	97	18	2	19	2	56
56	14	1	62	15	1	76	13	1	17	0	39
97	10	3	94	13	2	55	46	2	75	2	16
92	15	0	15	14	3	87	38	3	54	1	19
79	3	1	19	17	1	55	17	1	97	3	54

ALE AND BEER MEASURE.

(1)			(2)			(3)			(4)		
bar.	fir.	gal.	bar.	fir.	gal.	hhd.	gal.	qt.	hhd.	gal.	qt.
25	2	7	37	2	8	76	51	2	76	2	1
17	3	5	54	1	7	57	3	3	95	35	2
96	2	6	97	3	8	97	27	3	57	16	3
75	1	4	78	2	6	22	17	2	22	14	1
96	3	7	47	0	7	32	19	3	32	37	3
75	0	5	35	2	5	55	38	0	55	16	1

DRY MEASURE.

(1)			(2)			(3)			(4)		
qr.	bu.	p.	qr.	bu.	p.	ch.	bu.	p.	ch.	bu.	p.
75	7	2	36	2	1	75	27	2	73	2	1
37	2	3	71	0	3	57	3	1	41	24	1
51	2	0	53	6	0	95	25	3	92	16	1
79	7	1	82	4	1	76	35	2	70	13	2
55	0	3	95	3	3	97	25	1	54	17	3
96	2	1	78	2	1	75	16	3	79	25	1

TIME.

(1)		(2)			(3)			(4)		
m.	"	d.	h.	m.	w.	d.	h.	w.	d.	h.
57	35	72	23	27	71	3	11	57	2	15
16	27	54	15	35	51	2	9	95	3	21
58	45	97	13	31	76	0	21	76	0	15
18	31	58	21	45	95	3	21	53	2	21
35	21	96	20	48	79	1	15	98	2	18

THE APPLICATION.

man born in the year 1804, when was he 47 years of age?
Ans. 1851.

, *B*, *C*, *D*, went partners in the purchase of a quantity of goods; ut £7, half-a-guinea, and a crown, *B* 49*s.*, *C* 54*s.* 6*d.*, and *D* 87*d.* vas laid out in all? *Ans.* £13. 6*s.* 3*d.*

man lent his friend at different times these several sums, *viz:* 5. 15*s.*, £32. 7*s.*, £15. 14*s.* 10*d.*, and four score and nineteen , half a guinea, and a shilling. How much did he lend in all?
Ans. £236. 8*s.* 4*d.*

That is the estate worth *per annum*, when the taxes are 21 , the net income 8 score, £19. 14*s.*? *Ans.* £201. 15*s.*

here are three numbers; the first 215, the second 519, and the as much as the other two. What is the sum of them all?
Ans. 1468.

ought a parcel of goods for which I paid £54. 17*s.*, for packing ., carriage £1. 5*s.* 4*d.*, and spent about the bargain 14*s.* 3*d.* lo these goods stand me in? *Ans.* £57. 10*s.* 3*d.*

here are two numbers, the least whereof is 40, their difference desire to know what is the greater number, and the sum of
Ans. 54 *greater number*, 94 *sum.*

gentleman left his eldest daughter £1500 more than the st, and her fortune was 11 thousand 11 hundred and 11 pounds. vas the eldest sister's fortune, and what did the father leave
Ans. Eldest sister's fortune £13611. *Father left them* £25722.

RACTION OF MONEY, WEIGHTS, AND MEASURES.

. Commencing with the lowest denomination, subtract as in ; only when any subtractive quantity is greater than that above ow as many as make one of the next superior denomination, them to the upper quantity, from which take the less; set he difference, and carry one to the next higher denomination for ou borrowed.

F. *As in integers.*

MONEY.

(1)

	£.	s.	d.
Borrowed	715	2	$7\frac{1}{4}$
Paid	476	3	$8\frac{1}{2}$
Remains to pay ..	238	18	$10\frac{3}{4}$
Proof	715	2	$7\frac{1}{4}$

(2)

	£.	s.
Lent.....	316	3
Received ..	218	2

(3)

£.	s.	d.
87	2	10
79	3	$7\frac{1}{2}$

(4)

£.	s.	d.
3	15	$1\frac{1}{2}$
1	14	7

(5)

£.	s.	d.
25	2	$5\frac{1}{2}$
17	9	$8\frac{1}{2}$

(6)

£.	s.	d.
37	3	$4\frac{1}{4}$
27	5	$2\frac{1}{4}$

(7)

£.	s.	d.
321	17	$1\frac{1}{2}$
257	14	7

(8)

£.	s.	d.
59	15	$3\frac{1}{2}$
36	17	$2\frac{1}{2}$

(9)

£.	s.	d.
71	2	4
19	13	$7\frac{3}{4}$

(10)

£.	s.	d.
527	3	$5\frac{1}{2}$
139	5	$7\frac{1}{4}$

(11)

	£.	s.	d.
Borrowed	25107	15	7
Paid at different times	375	5	$5\frac{1}{4}$
	259	2	$7\frac{1}{2}$
	359	13	$4\frac{3}{4}$
	523	17	3
	274	15	$7\frac{1}{4}$
	325	13	5
Paid in all			
Remains to pay			

(12)

	£.	s.
Lent.....	250156	1
Received at several payments	271	13
	359	15
	475	13
	527	15
	272	16
	150	0

TROY WEIGHT.

(1)

	oz.	dt.	gr.
Bought.....	27	15	2
Sold.....	21	14	7
Unsold.....			

(2)

	oz.	dt.	gr.
	7	5	15
	6	7	14

(3)

	lb.	oz.	dt.	gr.
	52	1	7	2
	39	0	15	7

(4)

	lb.	oz.	dt.
	7	2	2
	5	7	1

AVOIRDUPOIS WEIGHT.

(1)			(2)			(3)			(4)			
lb.	oz.	dr.	lb.	oz.	dr.	cwt.	qrs.	lb.	ton.	cwt.	qrs.	lb.
25	11	15	35	10	5	35	1	21	21	1	2	7
17	9	13	29	12	7	25	1	20	9	1	3	5

APOTHECARIES WEIGHT.

(1)				(2)				(3)				(4)			
℥	ʒ	℈	gr.	℥	ʒ	℈	gr.	lb.	℥	ʒ	℈	lb.	℥	ʒ	℈
27	1	0	1	3	1	2	4	5	2	1	0	9	7	2	1
15	2	0	7	1	0	0	7	2	5	2	1	5	7	3	1

CLOTH MEASURE.

(1)			(2)			(3)			(4)		
E.E.	qrs.	n.	yds.	qrs.	n.	yds.	qrs.	n.	E.E.	qrs.	n.
35	2	2	47	1	0	71	1	2	35	2	1
17	2	1	35	2	2	3	2	1	14	3	2

LONG MEASURE.

(1)			(2)			(3)			(4)			
ft.	in.	bar.	yds.	ft.	in.	mi.	furl.	p.	lea.	mi.	furl.	p.
25	1	0	37	2	1	52	1	27	71	1	7	0
17	2	2	15	2	7	25	7	34	50	0	3	27

LAND MEASURE.

(1)			(2)			(3)			(4)		
a.	r.	p.	a.	r.	p.	a.	r.	p.	a.	r.	p.
75	1	27	37	1	27	25	0	1	325	2	1
39	0	27	35	2	15	17	1	0	379	3	5

WINE MEASURE.

(1)			(2)			(3)			(4)		
tun.	gal.	qts.	tier.	gal.	qts.	hhd.	gal.	qts.	tun.	hhd.	gal.
72	1	1	27	27	1	75	57	1	79	2	14
35	1	2	19	35	2	57	59	1	35	3	27

ALE AND BEER MEASURE.

(1)			(2)			(3)			(4)		
bar.	fir.	gal.	bar.	fir.	gal.	hhd.	gal.	qts.	hhd.	gal.	qts.
25	1	2	37	2	1	27	27	1	709	2	1
21	1	5	25	1	7	12	50	2	157	2	2

DRY MEASURE.

(1)			(2)			(3)			(4)		
qr.	bu.	p.	qr.	bu.	p.	ch.	bu.	p.	ch.	bu.	p.
72	1	2	65	2	1	79	3	0	35	3	3
35	2	3	57	2	3	54	7	1	23	5	1

TIME.

(1)			(2)			(3)			(4)		
h.	m.	″	d.	h.	m.	m.	w.	d.	m.	w.	d.
75	1	27	72	1	51	35	2	1	65	2	1
52	7	31	36	3	27	17	3	5	14	1	1

THE APPLICATION.

1. A man born in the year 1791, what was his age in year 1850? *Ans.* 59.

2. What is the difference between the age of a man born in 1809, and another born in 1854? *Ans.* 45.

3. A merchant had five debtors, *A*, *B*, *C*, *D*, and *E*, which together owed him £1156; *B*, *C*, *D*, and *E* owed him £737. What was *A's* debt? *Ans.* £419.

4. When an estate of £300 *per annum* is reduced, on payment of *taxes, to 12 score and £14. 6s.*, what is the tax? Ans. £45. 14s.

5. What is the difference between 9154, and the amount of 754 added to 305? *Ans.* 8095.

6. A horse in his furniture is worth £37. 5*s.*, out of it, 14 guineas. How much does the price of the furniture exceed that of the horse? *Ans.* £7. 17*s.*

7. A merchant, at his out-setting in trade, owed £750; he had in cash, commodities, the stocks, and good debts, £12510. 7*s.*; he cleared the first year by commerce £452. 3*s.* 6*d.* What was the net balance at 12 months' end? *Ans.* £12212. 10*s.* 6*d.*

8. A gentleman dying left £45247 between two daughters; the youngest was to have 15 thousand 15 hundred and twice 15 pounds. What was the eldest sister's fortune? *Ans.* £28717.

MULTIPLICATION OF SEVERAL DENOMINATIONS.

RULE. Multiply the first denomination by the number given, dividing the product by as many of that denomination as make one of the next, set down the remainder, and add the quotient to the next superior after it is multiplied.

If the given number is above 12, multiply by any two numbers, which, multiplied together, will make the same number; but if no two numbers multiplied together will make the exact number, then multiply the top line by as many as is wanting, adding it to the last product.

PROOF. By Division.

(1) £.	s.	d.	(2) £.	s.	d.	(3) £.	s.	d.	(4) £.	s.	d.
35	12	7¼	75	13	1½	62	5	4¼	57	2	4¾
		2			5			4			5
71	5	2¼									

(5) lb.	oz.	dwt.	gr.	(6) ton.	cwt.	qrs.	lb.	(7) yds.	qrs.	n.	(8) m.	fur.	p.
8	5	17	4	25	7	2	1	76	1	2	36	7	5
			7				8			2			5

(9) a.	r.	p.	(10) bar.	fir.	gal.	(11) bar.	fir.	gal.	(12) m.	fur.	p
7	2	1	32	1	7	26	2	7	54	2	1
		9			7			3			5

1. 18 yds. of cloth at 9*s.* 6*d.* per yd.

$9 \times 2 = 18$

		9
4	5	6
		2
8	11	0

2. 26lb. of tea at £1. 2*s.* 6*d.* per lb.

$8 \times 3 + 2 = 26$

		8
9	0	0
		3
27	0	0
Top line × 2 = 2	5	0
29	5	0

3. 21 ells of holland at 7*s.* 8½*d.* per ell. *Ans.* £8. 1*s.* 10½*d.*
4. 35 firkins of butter at 15*s.* 3½*d.* per firkin. *Ans.* £26. 15*s.* 2½*d.*
5. 75 lbs. of nutmegs at 7*s.* 2¾*d.* per lb. *Ans.* £27. 2*s.* 2¼*d.*
6. 37 yards of tabby at 9*s.* 7*d.* per yard. *Ans.* £17. 14*s.* 7*d.*
7. 97 cwt. of cheese at £1. 5*s.* 3*d.* per cwt. *Ans.* £122. 9*s.* 3*d.*
8. 43 dozen of candles at 6*s.* 4*d.* per dozen. *Ans.* £13. 12*s.* 4*d.*
9. 127 lbs. of bohea tea at 12*s.* 3*d.* per lb. *Ans.* £77. 15*s.* 9*d.*
10. 135 gallons of rum at 7*s.* 5*d.* per gallon. *Ans.* £50. 1*s.* 3*d.*
11. 74 ells of diaper at 1*s.* 4½*d.* per ell. *Ans.* £5. 1*s.* 9*d.*
12. 6 dozen pair of gloves at 1*s.* 10*d.* per pair. *Ans.* £6. 12*s.*

When the given number has ½ or ¼ connected with it, divide the price by 2, 4; when ¾, divide the price by 2, and that quotient by 2, and add both the results to the product of the quantity given.

13. 25½ ells of holland at 3*s.* 4½*d.* per ell.

$5 \times 5 = 25$

	5	
16	10½	
	5	
4	4	4½ = price of 25
0	1	8¼ = price of ½
4	6	0¾ = price of 25½

14. 75½ ells of diaper at 1*s.* 3*d.* per ell. *Ans.* £4. 14*s.* 4½*d.*
15. 19½ ells of damask at 4*s.* 3*d.* per ell. *Ans.* £4. 2*s.* 10½*d.*
16. 35½ ells of dowlas at 1*s.* 4*d.* per ell. *Ans.* £2. 7*s.* 4*d.*
17. 7¼ cwt. of Malaga raisins at £1. 1*s.* 6*d.* per cwt. *Ans.* £7. 15*s.* 10½*d.*
18. 6½ barrels of herrings at £3. 15*s.* 7*d.* per barrel. *Ans.* £24. 11*s.* 3½*d.*
19. 35½ cwt. of double refined sugar at £4. 15*s.* 6*d.* per cwt. *Ans.* £169. 10*s.* 3*d.*
20. 154½ cwt. of tobacco at £4. 17*s.* 10*d.* per cwt. *Ans.* £755. 15*s.* 3*d.*
21. 117¼ gallons of arrack at 12*s.* 6*d.* per gallon. *Ans.* £73. 5*s.* 7½*d.*
22. *85¾ cwt. of cheese at* £1. 7*s.* 8*d.* per cwt. Ans. £\18. 12*s.* 5*d.*

23. 29¼ lb. of fine hyson tea at £1. 3*s*. 6*d*. per lb.
Ans. £34. 7*s*. 4½*d*.
24. 17¾ yards of superfine scarlet drab at £1. 3*s*. 6*d*. per yard.
Ans. £20. 17*s*. 1½*d*.
25. 37½ yards of rich brocaded silk at 12*s*. 4*d*. per yard.
Ans. £23. 2*s*. 6*d*.
26. 56¾ cwt. of sugar at £2. 18*s*. 7*d*. per cwt. *Ans.* £166. 4*s*. 7½*d*.
27. 96½ cwt. of currants at £2. 15*s*. 6*d*. per cwt.
Ans. £267. 15*s*. 9*d*.
28. 45¾ lbs. of Belladine silk at 18*s*. 6*d*. per lb. *Ans.* £42. 6*s*. 4½*d*.
29. 87 bushels ¾ of wheat at 4*s*. 3*d*. per bushel. *Ans.* £18. 12. 11½*d*.
30. 120¾ cwt. of hops at £4. 7*s*. 6*d*. per cwt. *Ans.* £528. 5*s*. 7½*d*.

THE APPLICATION.

1. What sum of money must be divided amongst 18 men, so that each may receive £14. 6*s*. 8½*d*. ? *Ans.* £258. 0*s*. 9*d*.
2. A privateer of 250 men took a prize which amounted to £125. 15*s*. 6*d*. to each man, what was the value of the prize ?
Ans. £31443. 15*s*. 0*d*.
3. What is the difference between six dozen dozen and half a dozen dozen ; and what is their sum and product ?
Ans. Diff. 792, *Sum* 936, *Product* 62208.
4. What difference is there between twice eight and fifty and twice fifty-eight ; and what is their product ? *Ans.* 50 *Diff.*, 7656 *Product.*
5. There are two numbers, the greater of them is 37 times 45, and their difference is 19 times 4 ; their sum and product are required ?
Ans. 3254 *Sum,* 2645685 *Product.*
6. The sum of two numbers is 360, the less of them 144 ; what is their product, and the square of their difference ?
Ans. 31104 *Product,* 5184 *Square of their difference.*
7. In an army, consisting of 187 squadrons of horse, each 157 men, and 207 battalions, each 560 men, how many effective soldiers, supposing that in seven hospitals there are 473 sick ? *Ans.* 144806.
8. What sum did that gentleman receive in dowry with his wife, whose fortune was her wedding suit : her petticoat having two rows of furbelows, each furbelow 87 quills, and each quill 21 guineas ?
Ans. £3836. 14*s*.
9. A merchant had £19118 to begin trade with ; for 5 years together he cleared £1086 a year ; the next 4 years he made good £2715. 10*s*. 6*d*. a year ; but the last 3 years he was in trade had the misfortune to lose, one year with another, £475. 4*s*. 6*d*. a year ; what was his real fortune at 12 years' end ? *Ans.* £33984. 8*s*. 6*d*.

DIVISION OF SEVERAL DENOMINATIONS.

RULE. Divide the first denomination on the left hand; and, if any remains, multiply them by as many of the next less as make one of that, which add to the next, and divide as before.

PROOF. By Multiplication.

(1)

£.	s.	d.
2)25	2	4
12	11	2

(2)

£.	s.	d.
3)37	7	7

(3)

£.	s.	d.
4)57	5	7

(4)

£.	s.	d.
5)52	7	3

(5)

lb.	oz.	dwt.	gr.
7)75	3	7	5

(6)

lb.	oz.	dr.
8)35	14	13

(7)

ton.	cwt.	qrs.	lb.
6)5	10	1	13

(8)

yds.	qrs.	n.
11)35	1	3

(9)

m.	f.	p.
9)76	3	27

(10)

yds.	ft.	in.
12)75	2	9

(11)

bar.	fir.	gal.
12)35	2	5

(12)

bar.	fir.	gal.
9)55	3	7

(13)

ch.	bu.	p.
11)357	2	1

THE APPLICATION.

1. If a man spends £257. 2*s*. 5*d*. in 12 months' time, what is that per month? *Ans.* £21. 8*s*. 6½*d*.

2. The clothing of 35 charity boys came to £57. 3*s*. 7*d*, What is the expense of each? *Ans.* £1. 12*s*. 8*d*.

3. If I give £37. 6*s*. 4½*d*. for 9 pieces of cloth, what did I give per piece? *Ans.* £4. 2*s*. 11*d*.

4. If 20 cwt. of tobacco came to £27. 5*s*. 4½*d*., at what rate is that per cwt? *Ans.* £1. 7*s*. 3*d*.

5. What is the value of 1 hogshead of beer, when 120 are sold for £154. 17*s*. 10*d*.? *Ans.* £1. 5*s*. 9¾*d*.

6. Bought 72 yards of cloth for £85. 6*s*., I desire to know at what rate per yard? *Ans.* £1. 3*s*. 8½*d*.

7. *Gave £275. 3s. 4d.* for 36 bales of cloth. What is that for 2 bales? Ans. £15. 5*s*. 8¾*d*.

8. A prize of £7257. 3*s.* 6*d.* is to be equally divided amongst 500 sailors. What is each man's share? *Ans.* £14. 10*s.* 3½*d.*

9. There are 2545 bullocks to be divided among 509 men. I desire to know how many each man had, and the value of each man's share, supposing every bullock worth £9. 14*s.* 6*d.*

Ans. 5 bullocks each man, £48. 12*s.* 6*d. each share.*

10. A gentleman has a garden walled in, containing 9625 yards; the breadth was 35 yards. What was the length? *Ans.* 275.

11. A club in London, consisting of 25 gentlemen, joined for a lottery ticket of £10 value, which came up a prize of £4000. I desire to know what each man contributed, and what each man's share came to?

Ans. each contributed 8*s.*, *each share* £160.

12. A trader cleared £1156, equally in 17 years. How much did he lay by in a year? *Ans.* £68.

13. Another cleared £2805 in 7½ years. What was his yearly increase of fortune? *Ans.* £374.

14. What number added to the 43rd part of 4429 will raise it to 240? *Ans.* 137.

15. Divide 20*s.* between *A*, *B*, and *C*, in such sort that *A* may have 2*s.* less than *B*, and *C* 2*s.* more than *B*.

Ans. A 4*s.* 8*d.*, *B* 6*s.* 8*d.*, *C* 8*s.* 8*d.*

16. If there are 1000 men to a regiment, and but 50 officers, how many private men are there to one officer? *Ans.* 19.

17. What number is that which multiplied by 7847 will make the product 3013248? *Ans.* 384.

18. The quotient is 1083, the divisor 28604, what was the dividend, if the remainder came out 1788? *Ans.* 30979920.

BILLS OF PARCELS.

HOSIER'S.

Mr. John Thomas,

Bought of Samuel Green, March 7, 1854.

		s.	*d.*		
8 Pair of Worsted Stockings ..	at	4	6	per pair,	£
5 Pair of Thread ditto ..	at	3	2	,,	..
3 Pair of Black Silk ditto ..	at	14	0	,,	..
6 Pair of Milled Hose	at	4	2	,,	..
4 Pair of Cotton ditto	at	7	6	,,	..
2 Yards of Fine Flannel	at	1	8	,,	..

£7 12

MERCER'S.

Mr. Isaac Grant,

Bought of John Sims, March 12, 1854.

		s.	d.		
15 Yards of Satin	at	9	6	per yard,	£
18 Yards of Flowered Silk	at	17	4	,,	..
12 Yards of Rich Brocade	at	19	8	,,	..
16 Yards of Sarsenet	at	3	2	,,	..
13 Yards of Genoa Velvet	at	27	6	,,	..
23 Yards of Lutestring	at	6	3	,,	..

£62 2 5

LINEN-DRAPER'S.

Mr. Simon Surety,

Bought of Josiah Short, March 27, 1854.

		s.	d.		
4 Yards of Cambric	at	12	6	per yard,	£
12 Yards of Muslin	at	8	3	,,	..
15 Yards of Printed Linen	at	5	4	,,	..
2 Dozen of Napkins	at	2	3	each	....
14 Ells of Diaper	at	1	7	per ell	..
35 Ells of Dowlas	at	1	1½	,,	....

£17 4 6½

MILLINER'S.

Mrs. Bright,

Bought of Lucy Brown, April 5, 1854.

		s.	d.		
18 Yards of Fine Lace	at	12	3	per yard,	£
5 Pair of Fine Kid Gloves	at	2	2	per pair	..
12 Fans of French Mounts	at	3	6	each	....
2 Fine Laced Tippets	at	3	3	,,	....
4 Dozen Irish Lamb	at	1	3	per pair	..
6 Set of Knots	at	2	6	per set	..

£23 14 4

WOOLLEN-DRAPER'S.

Mr. Thomas Sage,

Bought of Ellis Smith, April 7, 1854.

		£.	s.	d.		
17	Yards of Fine Serge.....	at 0	3	9	per yard,	£
18	Yards of Drugget.....	at 0	9	0	,,	
15	Yards of Superfine Scarlet,	at 1	2	0	,,	
16	Yards of Black	at 0	18	0	,,	
25	Yards of Shalloon	at 0	1	9	,,	
17	Yards of Drab	at 0	17	6	,,	

£59 5 0

LEATHER-SELLER'S.

Mr. Giles Harris,

Bought of Abel Smith, April 15, 1854.

		s.	d.		
27	Calf-skins	at 3	9	per skin,	£
75	Sheep ditto	at 1	7	,,	..
36	Coloured ditto.....	at 1	8	,,	..
15	Buck ditto	at 11	6	,,	..
17	Russia Hides	at 10	7	each	....
120	Lamb-skins	at 1	2½	,,	....

£38 17 5

GROCER'S.

Mr. Richard Groves,

Bought of Francis Elliot, April 21, 1854.

		s.	d.		
25	lb. of Lump Sugar.....	at 0	6½	per lb.,	£
2	Loaves of Double Refined, weight 15 lb.	at 0	11½	,,	..
14	lb. of Rice	at 0	3	,,	..
28	lb. of Malaga Raisins	at 0	5	,,	..
15	lb. of Currants	at 0	5½	,,	..
7	lb. of Black Pepper	at 1	10	,,	..

£3 2 9½

CHEESEMONGER'S.

Mr. Charles Cross,

Bought of Samuel Grant, April 23, 1854

			s.	d.		
8	lb. of Cambridge Butter.....	at	0	6	per lb.,	£
17	lb. of New Cheese	at	0	4	,,	..
½	Firkin of Butter, weight 28 lb...	at	0	5½	,,	..
5	Cheshire Cheeses, weight 127 lb.	at	0	4	,,	..
2	Warwickshire do., weight 15 lb.	at	0	3	,,	..
12	lb. of Cream Cheese	at	0	6	,,	..

£3 14

CORN-CHANDLER'S.

Mr. Abraham Doyley,

Bought of Isaac Jones, April 29, 1854.

			s.	d.		
Tares..	19 Bushels	at	1	10	per bushel,	£
Peas ..	18 Bushels	at	3	9½	,,	..
Malt ..	7 Quarters.....	at	25	0	per quarter	
Hops ..	15 lb.	at	1	5	per lb.	
Oats ..	6 Quarters.....	at	2	4	per bushel..	
Beans..	12 Bushels	at	4	8	,,	..

£18 9

REDUCTION.

REDUCTION is the bringing or reducing quantities of one denomina into other quantities of another denomination, retaining the same va and is performed by multiplication and division.

First.—High denominations are brought into lower, by multiply by so many of the less as make one of the greater.

Secondly.—Lower denominations are brought into higher, by divid by so many of the less as make one of the greater.

REDUCTION TO LOWER DENOMINATIONS.

1. In £8, how many shillings and pence?

20

160 shillings.
12

1920 pence.

2. In £12, how many shillings, pence, and farthings?
Ans. 240*s.* 2880*d.* 11520 *far.*
3. How many shillings, pence, and farthings, are there in £18?
Ans. 360*s.* 4320*d.* 17280 *far.*
4. Reduce £7 and a crown into shillings and pence.
Ans. 145*s.* 1740*d.*
5. How many farthings are there in 21 guineas? *Ans.* 21168.
6. In £17. 5*s.* 3¼*d.*, how many farthings? *Ans.* 16573.
7. In £25. 14*s.* 1*d.*, how many shillings and pence?
Ans. 514*s.* 6169*d.*
8. In 15 crowns, how many shillings and sixpences?
Ans. 75*s.*, 150 *sixpences.*
9. How many crowns and shillings in £25? *Ans.* 100 *crowns*, 500*s.*
10. In 57 half-crowns, how many pence and farthings?
Ans. 1710*d.*, 6840 *farthings.*
11. In 52 crowns, as many half-crowns, shillings, and pence, how many farthings? *Ans.* 21424.
12. How many half-crowns, sixpences, and threepences, are there in £75? *Ans.* 600 *half-crowns*, 3000 *sixpences*, 6000 *threepences.*

REDUCTION TO HIGHER DENOMINATIONS.

13. In 1920 pence, how many shillings and pounds?

12)1920

2|0) 16|0 shillings.

£8 *Ans.* 160*s.*, £8.

14. In 11520 farthings, how many pence, shillings, and pounds?
Ans. 2880*d.*, 240*s.*, £12.
15. How many pence, shillings, and pounds, are there in 17280 farthings? *Ans.* 4320*d.*, 360*s.*, £18.
16. Reduce 1740 pence into shillings and pounds. *Ans.* 145*s.*, £7. 5*s.*
17. How many guineas in 21168 farthings? *Ans.* 21 *guineas.*
18. In 16573 farthings, how many pounds? *Ans.* £17. 5*s.* 3¼*d.*
19. In 6169 pence, how many shillings and pounds?
Ans. 514*s.*, £25. 14*s.* 1*d.*
20. In 900 pence, how many shillings and crowns?
Ans. 75*s.*, 15 *crowns.*
21. How many crowns and pounds in 500 shillings?
Ans. 100 *crowns*, £25.
22. In 6840 farthings, how many pence and half-crowns?
Ans. 1710*d.*, 57 *half-crowns.*
23. In 21424 farthings, how many crowns, half-crowns, shillings, and pence, and of each an equal number? *Ans.* 52.
24. How many sixpences, half-crowns, and pounds, in 6000 threepences? *Ans.* 3000 *sixpences*, 600 half-crowns, £75.

25. In 1560 pence, how many crowns and shillings?

6|0)156|0

26
5

130

Ans. 26 *crown*

26. Reduce 130 shillings into crowns and pence.

Ans. 26 *crowns,*

27. How many shillings, crowns, and pounds, in 60 guineas?

Ans. 1260*s.*, 252 *crown*

28. In £63, how many crowns, shillings, and guineas?

Ans. 252 *crowns,* 1260*s.*, 60 *g*

29. Reduce 76 moidores (each worth 27*s.*) into shillings and

Ans. 2052*s.*, £1(

30. Reduce £102. 12*s.* into shillings and moidores.

Ans. 2052*s.*, 76 *m*

31. How many shillings, half-crowns, and crowns, are there i and of each an equal number? *Ans.* 1308 *of each, and*

32. In 1308 half-crowns, as many crowns and shillings, ho pounds? *Ans.* £5

33. Seven men bought £15. 10*s.* each into the mint, to be c for guineas. How many must they have in all?

Ans. 103 *guineas,*

34. If 103 guineas and 7 shillings are to be divided amongst how many pounds sterling is that to each? *Ans.* £

35. A certain person had 25 purses, and in each purse 12 gu crown, and a moidore (worth 27*s.*), how many pounds sterling in all? *Ans*

36. A gentleman in his will leaves £50 to the poor, and order ½ should be given to ancient men, each to have 5*s.*; ¼ to poor each to have 2*s.* 6*d.*; ⅛ to poor boys, each to have 1*s.*; ⅛ girls, each to have 9*d.*, and the remainder to the person t tributed it. I demand how many of each sort there were, an the person that distributed the money had for his pains?

Ans. 66 *men,* 100 *women,* 200 *boys,* 222 *girls,* £2. 13*s.* 6*d. to the*

TROY WEIGHT.

37. In 27 ounces of gold, how many grains? *Ans.*

38. In 12960 grains of gold, how many ounces? *A*

39. In 3 lb. 10 oz. 7 dwt. 5 gr., how many grains? *Ans.*

40. In 8 ingots of silver, each weighing 7 lb. 4 oz. 17 dwt. *how many ounces,* pennyweights, and grains?

Ans. 711 oz., 14221 dwt., 34

41. How many ingots of 7 lb. 4 oz. 17 dwt. 15 gr. each, are there in 341304 grains? *Ans.* 8 *ingots.*

42. Bought 7 ingots of silver, each containing 23 lb. 5 oz. 7 dwt., how many grains? *Ans.* 945336.

43. A gentleman sent a tankard to his goldsmith that weighed 50 oz. 8 dwt., and ordered him to make it into spoons, each to weigh 2 oz. 16 dwt. How many had he? *Ans.* 18.

44. A gentleman delivered to a goldsmith 137 oz. 6 dwt. 9 gr. of silver, and ordered him to make it into tankards of 17 oz. 15 dwt. 10 gr. each, spoons of 21 oz. 11 dwt. 13 gr. per doz., salts of 3 oz. 10 dwt. each, and forks of 21 oz. 11 dwt. 13 gr. per dozen; and for every tankard to have one salt, a dozen of spoons, and a dozen of forks. What is the number of each he must have?

Ans. Two of each sort, 8 *oz.* 9 *dwt.* 9 *gr. over.*

AVOIRDUPOIS WEIGHT.

NOTE.—There are several sorts of silk which are weighed by a great pound of 24 oz., others by the common pound of 16 oz., therefore,

To bring great pounds into common, multiply by 8, and divide by 2, or add one-half.

To bring small pounds into great, multiply by 2, and divide by 3, or subtract one-third.

Things bought and sold by the Tale.

Dozens.	*Paper and Parchment.*
12 Pieces or things = 1 dozen.	24 Sheets ..... = 1 quire.
12 Dozen ..... = 1 gross.	20 Quires ..... = 1 ream.
12 Gross, or 144 Dozen .... = 1 great gross.	2 Reams ..... = 1 bundle.
	1 Doz. of Parchment = 12 skins.
	12 Skins ..... = 1 roll.

45. In 14769 ounces, how many cwt.? *Ans.* 8 *cwt.* 0 *qr.* 27 *lb.* 1 *oz.*

46. Reduce 8 cwt. 0 qr. 27 lb. 1 oz. into quarters, pounds, and ounces. *Ans.* 32 *qrs.* 923 *lb.* 14769 *oz.*

47. Bought 32 bags of hops, each 2 cwt. 1 qr. 14 lb., and another of 150 lb. How many cwt. in the whole? *Ans.* 77 *cwt.* 1 *qr.* 10 *lb.*

48. In 34 ton 17 cwt. 1 qr. 19 lb., how many pounds? *Ans.* 78111 *lb.*

49. In 350 great pounds, how many common? *Ans.* 525 *lb.*

50. In 27 cwt. of raisins, how many parcels of 18 lb. each? *Ans.* 168.

51. In 9 cwt. 2 qrs. 14 lb. of indigo, how many pounds? *Ans.* 1078.

52. In 547 great pounds, how many common pounds?

Ans. 820 *com. lb.* 8 *oz.*

53. Bought 27 bags of hops, each 2 cwt. 1 qr. 15 lb., and 1 bag of 137 lb. How many hundreds in the whole? *Ans.* 65 cwt. 2 qrs. 10 lb.

54. How many pounds in 27 hogsheads of tobacco, each weighing net 8¼ cwt.? *Ans.* 2646

55. In 552 common pounds of silk, how many great pounds?
Ans. 368.

56. How many parcels of sugar of 16 lb. 2 oz. are there in 16 cwt. 1 qr. 15 lb.? *Ans.* 113 *par.* 12 *lb.* 14 *oz.*

The allowances usually made in this weight, are TARE, TRET, *and* CLOFF.

TARE is an allowance made to the buyer for the weight of the box, barrel, bag, &c., which contains the goods bought, and is either
At so much per box, bag, barrel, &c.,
At so much per cwt., or,
At so much in the gross weight.

TRET is an allowance of 4 lb. in every 104 lb., or 1 lb. in 26 lb., for waste, dust, &c., made by the merchant to the buyer.

CLOFF is an allowance of 2 lb. in every 3 cwt. on some sort of goods.

GROSS WEIGHT is the whole weight of any sort of goods, and that which contains it.

SUTTLE is what remains when part of the allowance is deducted from the gross.

NETT is the pure weight, when all allowances are deducted.

RULE 1st. When the tare is at so much per bag, barrel, &c., multiply the number of bags, barrels, &c. by the tare, and subtract the produce from the gross, the remainder is nett.

NOTE.—To reduce pounds into gallons, multiply by 2, divide by 15.

57. In 7 frails of raisins, each weighing 5 cwt. 2 qrs. 5 lb. gross, tare at 23 lb. per frail, how much nett weight? *Ans.* 37 *cwt.* 1 *qr.* 14 *lb.*

lb.	cwt.	qrs.	lb.		cwt.	qrs.	lb.
23	5	2	5	or thus:	5	2	5
7			7	Subtract			23
qr. cwt. qr.							
28)161(5=1 1	38	3	7	gross.	5	1	10
140	1	1	21	tare.			7
21 lb.	37	1	14		37	1	14

58. In 241 barrels of figs, each 3 qrs. 19 lb. gross, tare 10 lb. per barrel, how many pounds nett? *Ans.* 22413.

59. What is the nett weight of 25 hogsheads of tobacco, weighing gross 163 cwt. 2 qrs. 15 lb., tare 100 lb. per hogshead?
Ans. 141 *cwt.* 1 *qr.* 7 *lb.*

60. In 16 bags of pepper, each 85 lb. 4 oz. gross, tare per bag 3 lb. 5 oz., how many pounds nett? *Ans.* 1311.

RULE 2nd. When the tare is at so much in the whole gross weight, subtract the given tare from the gross, the remainder is nett.

61. What is the nett weight of 5 hogsheads of tobacco, weighing gross *75 cwt. 1 qr.* 14 lb., tare in the whole 752 lb.?
Ans. 68 cwt. 2 qrs. 18 lb.

75 barrels of figs, each 2 qrs. 27 lb. gross, tare in the whole
ow much nett weight? *Ans.* 50 *cwt.* 1 *qr.*

. When the tare is at so much per cwt., divide the gross
r the aliquot parts of an cwt., which subtract from the gross,
nder is nett.

—7 lb. is $\frac{1}{16}$; 8 lb. is $\frac{1}{14}$; 14 lb. is $\frac{1}{8}$; 16 lb. is $\frac{1}{7}$; consequently,
tare is 7 lb., you must divide the gross weight by 16; when
by 14; when it is 14 lb., by 8; and when it is 16 lb., by 7.

hat is the nett weight of 18 butts of currants, each weighing
rs. 5 lb. tare, at 14 lb. per cwt.?

cwt.	qrs.	lb.	
8	2	5	
		9 × 2 = 18	
76	3	17	
		2	
8)153	3	6	gross weight.
19	0	25¼	tare, to be subtracted.
134	2	8¾	nett weight.

25 barrels of figs, each 2 cwt. 1 qr. gross, tare per cwt. 16 lb.,
h nett weight? *Ans.* 48 *cwt.* 0 *qr.* 24 *lb.*
hat is the nett weight of 9 hogsheads of nutmegs, each weighing
rt. 3 qrs. 14 lb., tare 16 lb. per cwt.? *Ans.* 68 *cwt.* 1 *qr.* 24 *lb.*
hat is the nett weight of 12 casks of argol, 84 cwt. 2 qrs. 14 lb.
lb. per cwt. tare? *Ans.* 74 *cwt.* 0 *qr.* 5¼ *lb.*

. When tret is allowed with tare, divide the pounds suttle
ie quotient is the tret, which subtract from the suttle, the
r is nett.

one butt of currants, weighing 12 cwt. 2 qr. 24 lb. gross, tare
cwt., tret 4 lb. per 104 lb., how many pounds nett?

cwt.	qrs.	lb.	
12	2	24	
4			
50			
28			
8)1424			gross weight in lbs.
178			tare.
26)1246			suttle.
47			tret.
1199			net weight in lbs.

68. In 7 cwt. 3 qrs. 27 lb. gross, tare 36 lb., tret 4 lb. per 104 lb how many pounds nett? *Ans.* 826 *l*

69. In 152 cwt. 1 qr. 3 lb. gross, tare 10 lb. per cwt., tret 4 lb. p 104 lb., how much nett weight? *Ans.* 133 *cwt.* 1 *qr.* 11 *l*

70. In 15 chests of sugar, weighing 117 cwt. 0 qr. 21 lb. gross, ta 173 lb., tret 4 lb. per 104 lb., how many cwt. nett?

Ans. 111 *cwt.* 0 *qr.* 22 *l*

RULE 5. When cloff is allowed, multiply the cwt. suttle by 2, divi the product by 3, the quotient will be the pounds cloff, which subtra from the suttle, the remainder will be nett.

71. What is the nett weight of 3 hogsheads of tobacco, weighin 15 cwt. 3 qrs. 20 lb. gross, tare 7 lb. per cwt., tret 4 lb. per 104 lb cloff 2 lb. for 3 cwt.? *Ans.* 14 *cwt.* 1 *qr.* 3 *l*

cwt.	qrs.	lb.	
16)15	3	20	gross.
0	3	27½	tare.
26)14	3	20½	suttle.
0	2	8	tret.
14	1	12½	suttle.
0	0	9½	cloff.
14	1	3	nett.

72. In 7 hogsheads of tobacco, each weighing gross 5 cwt. 2 qrs. 7 l tare 8 lb. per cwt., tret 4 lb. per 104 lb., cloff 2 lb. per 3 cwt., ho much nett weight? *Ans.* 34 *cwt.* 2 *qrs.* 8 *l*

APOTHECARIES WEIGHT.

73. 27 lb. 7 ℥ 2 ℥ 1 ℈ 2 gr., how many grains? *Ans.* 15902

74. How many lb. ℥, ℥, ℈, and gr. are there in 159022 grains?

Ans. 27 *lb.* 7 ℥ 2 ℥ 1 ℈ 2 *g*

CLOTH MEASURE.

75. In 27 yards, how many nails? *Ans.* 43

76. In 75 English ells, how many yards? *Ans.* 93 *yards*, 3 *q*

77. In 93¾ yards, how many English ells? *Ans.* 7

78. In 24 pieces, each containing 32 Flemish ells, how many Engli ells. *Ans.* 460 *ells*, 4 *q*

79. In 17 pieces of cloth, each 27 Flemish ells, how many yards?

Ans. 344 yards, 1 *q*

80. Bought 27 pieces of English stuffs, each 27 ells, how many yards? *Ans.* 911 *yards*, 1 *qr.*

81. In 911¼ yards, how many English ells? *Ans.* 729.

82. In 12 bales of cloth, each 25 pieces, each 15 English ells, how many yards? *Ans.* 5625.

LONG MEASURE.

83. In 57 miles, how many furlongs and poles?
Ans. 456 *furlongs*, 18240 *poles.*

84. In 7 miles, how many feet, inches, and barley-corns?
Ans. 36960 *feet*, 443520 *inches*, 1330560 *barley-corns.*

85. In 18240 poles, how many furlongs and miles?
Ans. 456 *furlongs*, 57 *miles.*

86. In 72 leagues, how many yards? *Ans.* 380160.

87. In 380160 yards, how many miles and leagues?
Ans. 216 *miles*, 72 *leagues.*

88. If from London to York be accounted 50 leagues, I demand how many miles, yards, feet, inches, and barley-corns?
Ans. 150 *miles*, 264000 *yards*, 792000 *feet*, 9504000 *inches*, 28512000 *barleycorns.*

89. How many barley-corns will reach round the world, which is 360 degrees, each degree 69 miles and a half.
Ans. 4755801600 *barley-corns.*

LAND MEASURE.

90. In 27 acres, how many roods and perches?
Ans. 108 *roods*, 4320 *perches.*

91. In 4320 perches, how many acres? *Ans.* 27.

92. A person having a piece of ground, containing 37 acres 1 pole, has a mind to dispose of 15 acres, I desire to know how many perches he will have left? *Ans.* 3521.

93. There are 4 fields to be divided into shares of 75 perches each; the first field containing 5 acres; the second, 4 acres, 2 poles; the third, 7 acres 3 roods; and the fourth, 2 acres 1 rood; I desire to know how many shares are contained therein? *Ans.* 40 *shares*, 42 *perches.*

WINE MEASURE.

94. Bought 5 tuns of wine, how many gallons and pints?
Ans. 1260 *gallons*, 10080 *pints.*

95. In 10080 pints, how many tuns? Ans. 5.

96. In *5896 gallons* of claret, how many pipes and hogsheads, and of *each a like number?* *Ans.* 31 of each, 37 gallons over.

97. A gentleman ordered his butler to bottle off $\frac{3}{4}$ of a pipe of into quarts, and the rest into pints; I desire to know how many of each he had? *Ans.* 28 *dozen of*

ALE AND BEER MEASURE.

98. In 46 barrels of beer, how many pints? *Ans.* 1
99. In ten barrels of ale, how many gallons and quarts?
Ans. 360 *gallons*, 1440 q
100. In 72 hogsheads of beer, how many barrels? *Ans*
101. In 108 barrels of beer, how many hogsheads? *An*

DRY MEASURE.

102. In 120 quarters of wheat, how many bushels, pecks, ga and quarts?
Ans. 960 *bushels*, 3840 *pecks*, 7680 *gallons*, 30720 q
103. In 30720 quarts of corn, how many quarters? *Ans*
104. In 20 chaldron of coals, how many pecks? *Ans.*
105. In 273 lasts of corn, how many pecks? *Ans.* 8

TIME.

106. In 72015 hours, how many weeks?
Ans. 428 *weeks*, 4 *days*, 15
107. How many days is it since the birth of our Saviour to C mas, 1845? *Ans.* 67
108. Stowe writes that London was built 1108 years befor Saviour's birth, how many hours is it since to Christmas 1855, re ing 365 days 6 hours a year? *Ans.* 25973658
109. From November, 17, 1848, to September 12, 1849, how days? *Ans*
110. From July 18, 1849, to December 27 in the same year, many days? *Ans.* 162
111. From July 18, 1823, to April 18, 1850, how many yea days? *Ans.* 26 *years*, 9770

THE SINGLE RULE OF THREE DIRECT.

questions coming under this rule, *three* quantities are given to fourth.

the three given quantities, two must always be of the same kind, he remaining one must always be of the same kind as the quan- o be found. The given quantities and the quantity sought must ur form a proportion; so that the first of them must bear to the d the same relation (or ratio) that the third bears to the fourth. ule is this:—

LE. First state the question; that is, place the given quantities in , and in such order that the first and second may be of one kind, he third of the same kind as the quantity sought, taking care that latter quantity ought to be greater or less than the third quan- n the stating, so must the first be, in like manner, greater or less the second. Bring the first and second to the same denomination *lowest* denomination which appears in either, then, considering duced quantities to be mere *numbers*, multiply the third quantity e second number, and divide the product by the first: the quotient e the answer, or required fourth term of the proportion.

Examples.

If 4 lb. of sugar cost 1*s.* 6*d.*, what will 54½ lb. cost

	lb.	lb.	*s.*	*d.*	*d.*	£	*s.*	*d.*
g stating is	4	: 54½	:: 1	6	or 18	: 1	0	5¼
is the lowest denomina-	2	2						
in the first and second	—	—						
ities is half-pounds, you	8	109						
both these quantities		18						
half-pounds: the first		—						
erms then become the		872						
ers 8 : 109, and it only		109						
ns to multiply the 18*d.*		—						
09, and then to divide		8)1962 pence.						
: you thus get 245¼		—						
for the price of		12) 245¼ pence.						
lb.		—						
		Ans. 20*s.* 5¼*d.*						

If a gallon of ale cost 3*d.*, what is that per barrel? *Ans.* 9*s.*

If a pair of shoes cost 4*s.* 6*d.*, what will 12 dozen come to?
Ans. £32. 8*s.*

If 12 dozen pair of stockings cost £32. 8*s.*, what is that per pair?
Ans. 4*s.* 6*d.*.

If 1 *yard of cloth cost* 15*s.* 6*d.*, what will 32 yards cost at the *rate?*
Ans. £24. 16*s.*

6. If 32 yards of cloth cost £24. 16*s.*, what is the value of 1 yard?
Ans. 15*s.* 6*d.*

7. If 1 lb. of sugar cost 10½*d.*, what is the worth of 1 cwt.?
Ans. £4. 18*s.*

8. If I gave £4. 18*s.* for 1 cwt. of sugar, at what rate did I buy it at per lb.? *Ans.* 10½*d.*

9. If I buy 20 pieces of cloth, each 20 ells, for 12*s.* 6*d.* per ell, what is the value of the whole? *Ans.* £250.

10. Bought 20 pieces of Holland, each 20 ells, for £250, what is that per ell? *Ans.* 12*s.* 6*d.*

11. What will 25 cwt. 3 qrs. 14 lb. of tobacco come to, at 15½*d.* per lb.? *Ans.* £187. 3*s.* 3*d.*

12. I gave £187. 3*s.* 3*d.* for 25 cwt. 3 qrs. 14 lb. of tobacco, at what rate did I buy it at per lb.? *Ans.* 15½*d.*

13. Bought 27¼ yards muslin, at 6*s.* 9½*d.* per yard, what does it amount to? *Ans.* £9. 5*s.* 0$\frac{3}{4}$*d.* 2 *rem.*

14. Bought 17 cwt. 1 qr. 14 lb. of iron, at 3½*d.* per lb., what does it come to? *Ans.* £26. 7*s.* 0½*d.*

15. If coffee is sold for 5½*d.* per ounce, what must be given for 2 cwt.? *Ans.* £82. 2*s.* 8*d.*

16. How many yards of cloth may be bought for £21. 11*s.* 1½*d.*, when 3½ yards cost £2. 14*s.* 3*d.*? *Ans.* 27 *yds.* 3 *qrs.* 1 *nail*, 84 *rem.*

17. If 3½ lb. of Cheshire cheese cost 1*s.* 1*d.*, what cost 1 cwt.?
Ans. £1. 14*s.* 8*d.*

18. If 1 cwt. of Cheshire cheese cost £1. 14*s.* 8*d.*, what must I give for 3½ lb.? *Ans.* 1*s.* 1*d.*

19. Bought 1 cwt. 24 lb. 8 oz. of old lead, at 9*s.* per cwt., what does it come to? *Ans.* 10*s.* 11½*d.*, 896 *rem.*

20. If 1 cwt. 24 lb. 8 oz. of lead be worth 10*s.* 11½*d.* $\frac{112}{224}$, what is that per cwt.? *Ans.* 9*s.*

21. If a gentleman's income is £500 a year, and he spends 19*s.* 4*d.* per day, how much does he lay by at the year's end?
Ans. £147. 8*s.* 4*d.*

22. If I buy 14 yards of cloth for 10 guineas, how many Flemish ells can I buy for £283. 17*s.* 6*d.*, at the same rate?
Ans. 504 *Flemish ells*, 2 *quarters.*

23. If £283. 17*s.* 6*d.* will buy 504 Flemish ells, 2 quarters, what quantity of yards can I have for £10. 10*s.*? *Ans.* 14 *yards.*

24. If 504 Flemish ells, 2 quarters, cost £283. 17*s.* 6*d.*, at what rate must I give for 14 yards? *Ans.* £10. 10*s.*

25. Gave £1. 1*s.* 8*d.* for 3 lb. of coffee, what must be given for 29 lb. 4 oz.? *Ans.* £10. 11*s.* 8*d.*

26. Bought 29 lb. 4 oz. of coffee for £10. 11*s.* 3*d.*, what is the value of 3 lb.? *Ans.* £1. 1*s.* 8*d.*

27. If 1¼ oz. of coffee cost 6½*d.*, what will 3¼ oz. cost at the same rate? *Ans.* 1*s.* 1½*d.*, 1 *rem.*

28. If 1 English ell, 2 quarters, cost 4*s.* 7*d.*, what will 39½ yards cost at the same rate? *Ans.* £5. 3*s.* 5½*d.*

29. If 1 oz. of gold is worth £5. 4*s.* 2*d.*, what is the worth of 1 grain?
Ans. 2½*d.*, 200 *rem.*

30. If 14 yards of broadcloth cost £9. 12*s.*, what is the purchase of 75 yards? *Ans.* £51. 8*s.* 6¾*d.*, 6 *rem.*

31. If 27 yards of Holland cost £5. 12*s.* 6*d.*, how many ells English can I buy for £100? *Ans.* 384.

32. If 1 cwt. cost £12. 12*s.* 6*d.*, what must I give for 14 cwt. 1 qr. 19 lb.? *Ans.* £182. 0*s.* 11¼*d.*, 8 *rem.*

33. Bought 7 yards of cloth for 17*s.* 8*d.*, what must be given for 5 pieces, each containing 27¼ yards? *Ans.* £17. 7*s.* 0¼*d.*, 2 *rem.*

34. If 7 oz. 11 dwt. of gold be worth £35, what is the value of 14 lb. 9 oz. 12 dwt. 16 gr. at the same rate? *Ans.* £823. 9*s.* 3¾*d.*, 552 *rem.*

35. A draper bought 420 yards of broadcloth, at the rate of 14*s.* 10¾*d.* per ell English, how much did he pay for the whole? *Ans.* £250. 5*s.*

36. A gentleman bought a wedge of gold, which weighed 14 lb. 3 oz. 8 dwt., for the sum of £514. 4*s.*, at what rate did he pay for it per ounce? *Ans.* £3.

37. A grocer bought 4 hogsheads of sugar, each weighing neat 6 cwt. 2 qrs. 14 lb., which cost him £2. 8*s.* 6*d.* per cwt.; what is the value of the 4 hogsheads? *Ans.* £64. 5*s.* 3*d.*

38. A draper bought 8 packs of cloth, each containing 4 parcels, each parcel 10 pieces, and each piece 26 yards, and gave after the rate of £4. 16*s.* for 6 yards, I desire to know what the 8 packs stood him in? *Ans.* £6656.

39. If 24 lb. of raisins cost 6*s.* 6*d.*, what will 18 frails cost, each weighing neat 3 qrs. 18 lb. *Ans.* £24. 17*s.* 3*d.*

40. If 1 ounce of silver be worth 5*s.*, what is the price of 14 ingots, each weighing 7 lb. 5 oz. 10 dwt.? *Ans.* £313. 5*s.*

41. What is the price of a pack of wool weighing 2 cwt. 1 qr. 19 lb., at 8*s.* 6*d.* per stone? *Ans.* £8. 4*s.* 6¼*d.*, 10 *rem.*

42. Bought 59 cwt. 2 qrs. 24 lb. of tobacco, at £2. 17*s.* 4*d.* per cwt., what does it come to? *Ans.* £171. 3*s.* 7¼*d.*, 80 *rem.*

43. What is the half-year's rent of 547 acres of land, at 15*s.* 6*d.* per acre per annum? *Ans.* £211. 19*s.* 3*d.*

44. Bought 171 tons of lead, at £14 per ton; paid carriage and other incidental charges, £4. 10*s.*, I require the value of the lead, and what it stands me in per lb.?

Ans. £2398. 10*s.*—*value*, 1½*d. per lb.*, 4320 *rem.*

45. If a pair of stockings cost 3*s.* 4*d.*, how many dozen may I buy for £43. 5*s.*? *Ans.* 21 *doz.* 7½ *pair.*

46. Bought 27 doz. 5 lb. of candles, after the rate of 17*d.* per 3 lb., what did they cost me? *Ans.* £7. 15*s.* 4½*d.*, 1 *rem.*

47. If an ounce of fine gold is sold for £3. 10*s.*, what comes 7 ingots to, each weighing 3 lb. 7 oz. 14 dwt. 21 gr. at the same price?

Ans. £1071. 14*s.* 5½*d.*

48. If my horse stands me in 9½*d.* per day keeping, what will be the charge of 11 horses for the year? *Ans.* £158. 18*s.* 6½*d.*

49. A factor bought 86 pieces of stuff, which cost him £517. 19*s.* 4*d.*, at 4*s.* 10*d.* per yard, I demand how many yards there were, and how many ells English in a piece?

Ans. 2143½ *yards*, 14 *rem.*; *and* 19 ells, 4 quarters, 2 nails (64 *rem.*), *in a piece.*

50. A gentleman hath an annuity of £896. 17*s.* per annum. I desire to know how much he may spend daily, that at the year's end he may lay up 200 guineas, and give to the poor quarterly 27 half-sovereigns?

Ans. £1. 14*s.* 8*d.*, 44 *rem.*

THE RULE OF THREE INVERSE.

A QUESTION is said to belong to the Rule of Three Inverse when one of the three given quantities *increases* as another of them, which is usually not of the same kind, *diminishes*. Thus, if a certain amount of work is to be performed by labourers, then as the number of *men* is *increased* the *time* they are employed will be *diminished*. In like manner, if a sum of money is to be equally divided among a number of persons, the *greater* the number of persons the *less* will be the share of each. And again, if a stock of provisions be provided for a body of men, the fewer the men the longer it will last.

In considering a Rule of Three question, you will have, therefore, to pay attention to these particulars, and to discover from the nature of the question whether the sought *fourth term* of the proportion is to be greater or less than the given *third term*, and to arrange the given first and second terms accordingly, as in the preceding rule. Whether the question belong to the Rule of Three Direct, or to the Rule of Three Inverse, you may always proceed as follows:—

RULE 1. Write down the given *third* term of the stating, or proportion.

2. Then according as the required fourth term is to be greater or less than *it*, so let the *second* term of the stating be greater or less than the *first*; and proceed as in the foregoing examples.

Examples.

1. If 8 men can do a piece of work in 12 days, how many days can 16 men perform the same in?

Here the answer is to be days: so that putting the 12 days for the third term of the stating, you consider whether the answer will be *more* or *fewer* than 12 days. It is plain, that as there are more workmen the number of days will be fewer, so you take care that the *smaller* of the other two given numbers, 8 and 16, shall form the *second* term: the stating will, therefore, be—

				days.		days.
16	:	8	::	12	:	6

8

16)96(6 *days, the Ans.*
96

2. When a peck of wheat is sold for 2*s.*, if the penny loaf weighs 8 oz., how much should it weigh when the peck is worth but 1*s.* 6*d.*?

As the answer is to be *weight*, you put the 8 oz. for the third term, and since the weight should *increase* as the price of wheat *diminishes*, it is plain that the answer must be more than 8 oz., so you take care to put the *greater* of the two other given quantities, 2*s.* and 1*s.* 6*d.*, *second*: the stating is, therefore,—

1*s.* 6*d.*	2*s.*	::	8 oz.	:	10 oz. $10\frac{2}{3}$ dr.
2	2		4		
3 *sixpences.*	4		3)32		
			10 oz. 2		
			16		
			3)32		
			$10\frac{2}{3}$ dr.		

The Answer is 10 *ounces*, 10 *drams and two-thirds of a dram.*

3. If 55 men can build a house in 90 days, how many men can do the same in 50 days? *Ans.* 99 *men.*

4. How many pieces of money of 20*s.* value are equal to 240 pieces of 12*s.* each? *Ans.* 144.

5. How many yards of 3 quarters wide are equal in measure to 30 yards of 5 quarters wide? *Ans.* 50.

6. If I lend my friend £200 for 12 months, how long ought he to lend me £150 to requite my kindness? *Ans.* 16 *months.*

7. If for 24*s.* I have 1200 lb. carried 36 miles, how many pounds can I have carried 24 miles for the same money? *Ans.* 1800 *lb.*

8. If 108 workmen finish a piece of work in 12 days, how many are sufficient to finish it in 3 days? *Ans.* 432.

9. An army besieging a town, in which were 1000 soldiers, with provisions for 3 months, how many soldiers departed, when the provisions lasted them 6 months? *Ans.* 500.

10. If £20 worth of wine is sufficient to serve an ordinary of 100 men, when the tun is sold for £30, how many will £20 worth suffice, when the tun is sold but for £24? *Ans.* 125 *men.*

11. A courier makes a journey in 24 days, when the day is but 12 hours long, how many days will he be going the same journey, when the days are 16 hours long? *Ans.* 18 *days.*

12. How much plush is sufficient for a cloak which has in it 4 yards of 7 quarters wide of stuff for the lining, the plush being but 3 quarters wide? *Ans.* $9\frac{1}{3}$ *yards.*

13. If 14 pioneers make a trench in 18 days, of 12 hours, how many days will 34 take to do the same? *Ans.* 7 *days*, 4 *hours*, 56 *minutes.*

14. Borrowed of my friend £64 for 8 months, and he hath occasion another *time to borrow* of me for 12 months, how much must I lend him *to requite his former kindness* to me? Ans. £42. 13*s.* 4*d*

15. A regiment of soldiers, consisting of 1000 men, are to have new coats, each coat to contain 2¼ yards of cloth, 5 quarters wide, and to be lined with shalloon of 3 quarters wide: I demand how many yards of shalloon will line them? *Ans.* 4166 *yards*, 2 *quarters*, 2 *nails*, 2 *rem.*

THE DOUBLE RULE OF THREE.

THE Double Rule of Three is so called because the questions coming under that head would require, at least, two Single Rule of Three statings for the working of them, if the operations were to be performed according to the preceding directions. But the process may be abridged by help of the following rule:—

RULE 1. From carefully reading the question, find out what *kind* of quantity the answer is to be in, and write down for the *third* term that one of the given quantities which is of *the same kind.*

2. Take any two of the remaining given quantities, which are like in kind, without taking any notice of the other quantities, and complete the stating just as you would do if those other quantities were not given.

3. In like manner, take another pair of the given quantities, like in kind, and place them under the former pair, so arranging them that they and the third term, before written down, may form another Single Rule of Three stating, and, as before, take no notice of the remaining quantities.

4. Proceed in this way till each pair of like quantities has been written down under the first pair.

5. Multiply the third term by the product of all the terms in the second place, and divide the result by the product of all the terms in the first place, regarding the products here spoken of as mere numbers: the quotient will be the answer.

Examples.

1. If 14 horses eat 56 bushels of oats in 16 days, how many bushels will be sufficient for 20 horses for 24 days?

As the answer is to be in *bushels*, you take care that *bushels* shall be in the third term. You see, likewise, that there are two other pairs of quantities—namely, *days* and *horses;* you may take, first, either of these pairs you please, for the first and second terms: suppose you take *days*, then you will only have to consider this Single Rule of Three question:—If 56 bushels last 16 days, how many will last 24 days? Of course, there must be a *greater* number of bushels; so that your stating must be—

				bush.
16	:	24	::	56

Now, taking the other pair of like quantities—namely, horses—and taking no account of the days, you have then to consider *this* Single Rule of Three question, namely:—If 14 horses eat 56 bushels, how many bushels will 20 horses eat? It is plain that the answer must be a *greater* number of bushels, so that your stating would be—

				bush.
14	:	20	::	56

Therefore, the two statings, written one under the other, would stand thus—

				bush.
16	:	24	::	56
14	:	20	::	—

And by the rule, $56 \times 24 \times 20$ is to be divided by 16×14; so that the work is as follows:—

bush.
56
24

16 224
14 112

64 1344
16 20

224)26880(120 *bushels, the Ans.*
224

448
448

0

2. If 8 men can mow 112 acres of grass in 14 days, how many men must there be to mow 2000 acres in 10 days?

Here the answer is to be in *men*, therefore *men* must be in the third term. The 8 men being thus put down, you may take the acres for the first and second terms; and as 2000 acres require more men than 112, you put 112 for the first term, and 2000 for the second. Again, taking the days, still regarding the 8 men as the third term of the second stating, you see that if for 14 days 8 men are necessary, for only 10 days *more* men will be required; therefore, you put 10 days for the first term, and 14 for the second. The Double Rule of Three statings are, therefore, as below:—

				men.
112	:	2000	::	8
10	:	14	::	—

And you then work by the rule as follows:—

men.
8
2000
16000
14
64000
16

112
10
112,0)22400,0(200 *men, the Ans.*
224
00

3. If £100 in 12 months gain £6 interest, how much will £75 gain in 9 months? *Ans.* £3. 7*s.* 6*d.*

4. If a carrier receives £2. 2*s.* for the carriage of 3 cwt. 150 miles, how much ought he to receive for the carriage of 7 cwt. 3 qrs. 14 lb. for 50 miles? *Ans.* £1. 16*s.* 9*d.*

5. If a regiment of soldiers, consisting of 136 men, consume 351 quarters of wheat in 108 days, how many quarters of wheat will 11232 soldiers consume in 56 days? *Ans.* 15031.*

6. If 40 acres of grass be mowed by 8 men in 7 days, how many acres can be mowed by 24 men in 28 days? *Ans.* 480.

7. If 40*s.* will pay 8 men for 5 days' work, how much will pay 32 men for 24 days' work? *Ans.* £38. 8*s.*

8. If £100 in 12 months gain £6 interest, what principal will gain £3. 7*s.* 6*d.* in 9 months? *Ans.* £75.

9. If a regiment, consisting of 939 soldiers, consume 351 quarters of wheat in 168 days, how many soldiers will consume 1404 quarters in 56 days? *Ans.* 11268.

10. In a family consisting of 7 persons, there are drunk out 2 kilderkins of beer in 12 days, how many kilderkins will there be drunk out by another family of 14 persons in 8 days? *Ans.* 2 *kil.* 12 *gal.*

11. If the carriage of 60 cwt. 20 miles cost £14. 10*s.*, what weight can I have carried 30 miles for £5. 8*s.* 9*d.* at the same rate of carriage? *Ans.* 15 *cwt.*

12. If 2 horses eat 8 bushels of oats in 16 days, how many horses will eat up 3000 quarters in 24 days? *Ans.* 4000.

13. If £100 in 12 months gain £7 interest, what is the interest of £571 for 6 years? *Ans.* £239. 16*s.* 4$\frac{3}{4}$*d.*

14. If I pay 10*s.* for the carriage of 2 tons 6 miles, what must I pay for the carriage of 12 tons 17 cwt. 17 miles? *Ans.* £9. 2*s.* 0$\frac{1}{2}$*d.*

* Remainders are not inserted in the answers to these examples. They indicate *fractional parts*, and may be disregarded till FRACTIONS are fully understood.

PRACTICE

Is so called, from the general use thereof by all persons concerned in trade and business.

All questions in this rule are performed by taking aliquot or even parts, by which means many tedious reductions are avoided; the table of these aliquot parts is as follows:—

Of a Pound.			*Of a Shilling.*			*Of a Ton.*			*Of an Hundred.*		
s.	*d.*		*d.*			cwt.			qrs.	lb.	
10	0	is .. $\frac{1}{2}$	6	is .... $\frac{1}{2}$		10	is .... $\frac{1}{2}$		2 or	56 is .. $\frac{1}{2}$	
6	8	,, .. $\frac{1}{3}$	4	,, ... $\frac{1}{3}$		5	,, ... $\frac{1}{4}$		1 ,,	28 ,, .. $\frac{1}{4}$	
5	0	,, .. $\frac{1}{4}$	3	,, ... $\frac{1}{4}$		4	,, ... $\frac{1}{5}$			14 ,, .. $\frac{1}{8}$	
4	0	,, .. $\frac{1}{5}$	2	,, ... $\frac{1}{6}$		2½	,, ... $\frac{1}{8}$		*Of a Quarter.*		
3	4	,, .. $\frac{1}{6}$	1½	,, ... $\frac{1}{8}$		2	,, ... $\frac{1}{10}$		14	is ..... $\frac{1}{2}$	
2	6	,, .. $\frac{1}{8}$	1	,, ... $\frac{1}{12}$					7	,, ... $\frac{1}{4}$	
2	0	,, .. $\frac{1}{10}$							4	,, ... $\frac{1}{7}$	
1	8	,, .. $\frac{1}{12}$							3½	,, ... $\frac{1}{8}$	

NOTE. When, in the following rules, you are directed to "take aliquot parts," remember that you do not divide by $\frac{1}{4}$, $\frac{1}{12}$, &c.; but by 4, 12, &c.

RULE 1. When the price is less than a penny, take the aliquot parts that are in a penny; then divide by 12 and 20, it will be the answer.

$\frac{1}{4}d.$ is $\frac{1}{4}$ 5704 lb. at $\frac{1}{4}d.$

12)1426

2|0)11|8 10

£5. 18*s.* 10*d.*

7695 at $0\frac{1}{2}d.$
Ans. £16. 0*s.* $7\frac{1}{2}d.$

6547 at $0\frac{3}{4}d.$
Ans. £20. 9*s.* $2\frac{1}{4}d.$

5740 at $0\frac{1}{2}d.$
Ans. £11. 19*s.* 2*d.*

4573 at $0\frac{3}{4}d.$
Ans. £14. 5*s.* $9\frac{3}{4}d.$

RULE 2. When the price is less than a shilling, take the aliquot part or parts that are in a shilling, add them together and divide by 20, as before.

1*d.* is $\frac{1}{12}$ 7547 at 1*d.*

2|0)62|8 11

£31. 8*s.* 11*d.*

54325 at $1\frac{1}{4}d.$
Ans. £339. 10*s.* $7\frac{1}{2}$d.

6254 at $1\frac{3}{4}d.$
Ans. £45. 12*s.* $0\frac{1}{4}d.$

3250 at $2\frac{3}{4}d.$
Ans. £37. 4*s.* $9\frac{1}{4}d.$

2715 at 3*d.*
Ans. £33. 18*s.* 9*d.*

1*d.* is $\frac{1}{12}$ 3751 at $1\frac{1}{2}d.$

$\frac{1}{2}$ is $\frac{1}{2}$ 312 7
78 $1\frac{3}{4}$

2|0)39|0 $8\frac{3}{4}$

£19. 10s. $8\frac{3}{4}d.$

2351 at 2*d.*
Ans. £19. 11*s.* 10*d.*

7210 at $2\frac{1}{4}d.$
Ans. £67. 11*s.* $10\frac{1}{2}d.$

2710 at $2\frac{1}{2}d.$
Ans. £28. 4*s.* 7*d.*

7062 at $3\frac{1}{4}d.$
Ans. £95. 12*s.* $7\frac{1}{2}d.$

2147 at $3\frac{1}{4}d.$
Ans. £31. 6*s.* $2\frac{1}{4}d.$

7000 at $3\frac{3}{4}d.$
Ans. £109. 7*s.* 6*d.*

3257 at 4*d.* *Ans.* £54. 5*s.* 8*d.*	2708 at 6¾*d.* *Ans.* £76. 3*s.* 3*d.*	7924 at 9½*d.* *Ans.* £313. 13*s.* 2*d.*
2056 at 4½*d.* *Ans.* £36. 8*s.* 2*d.*	3271 at 7*d.* *Ans.* £95. 8*s.* 1*d.*	2150 at 9¾*d.* *Ans.* £87. 6*s.* 10½*d.*
3752 at 4½*d.* *Ans.* £70. 7*s.*	3254 at 7¼*d.* *Ans.* £98. 5*s.* 11½*d.*	6325 at 10*d.* *Ans.* £263. 10*s.* 10*d.*
2107 at 4¾*d.* *Ans.* £41. 14*s.* 0¼*d.*	2701 at 7½*d.* *Ans.* £84. 8*s.* 1½*d.*	5724 at 10¼*d.* *Ans.* £244. 9*s.* 3*d.*
3210 at 5*d.* *Ans.* £66. 17*s.* 6*d.*	3714 at 7¾*d.* *Ans.* £119. 18*s.* 7½*d.*	6327 at 10¼*d.* *Ans.* £270. 4*s.* 3¾*d.*
2715 at 5¼*d.* *Ans.* £59. 7*s.* 9¾*d.*	2710 at 8*d.* *Ans.* £90. 6*s.* 8*d.*	3254 at 10½*d.* *Ans.* £142. 7*s.* 3*d.*
3120 at 5½*d.* *Ans.* £71. 10*s.*	3514 at 8¼*d.* *Ans.* £120. 15*s.* 10½*d.*	7291 at 10¾*d.* *Ans.* £326. 11*s.* 6¼*d.*
7521 at 5¾*d.* *Ans.* £180. 3*s.* 9¾*d.*	2759 at 8½*d.* *Ans.* £97. 14*s.* 3½*d.*	3256 at 11*d.* *Ans.* £149. 4*s.* 8*d.*
3271 at 6*d.* *Ans.* £81. 15*s.* 6*d.*	9872 at 8¾*d.* *Ans.* £359. 18*s.* 4*d.*	7254 at 11¼*d.* *Ans.* £340. 0*s.* 7½*d.*
7914 at 6¼*d.* *Ans.* £206. 1*s.* 10½*d.*	5272 at 9*d.* *Ans.* £197. 14*s.*	3754 at 11½*d.* *Ans.* £179. 17*s.* 7*d.*
3250 at 6½*d.* *Ans.* £88. 0*s.* 5*d.*	6325 at 9½*d.* *Ans.* £243. 15*s.* 6¼*d.*	7972 at 11¾*d.* *Ans.* £390. 5*s.* 11*d.*

RULE 3. When the price is more than one shilling, and less than two, take the part or parts for so much of the given price as is more than a shilling, which add to the given quantity, and divide by 20, it will give the answer.

¼ is $\frac{1}{4}$ 2106 at 12¼*d.* 43 10¼ 2|0)214|9 10¼ £107. 9*s.* 10¼*d.*	2712 at 12¾*d.* *Ans.* £144. 1*s.* 6*d.*	7904 at 1*s.* 1¾*d.* *Ans.* £452. 16*s.* 8*d.*
	2107 at 1*s.* 1*d.* *Ans.* £114 2*s.* 7*d.*	3750 at 1*s.* 2*d.* *Ans.* £218. 15*s.* 0*d.*
¼ is $\frac{1}{24}$ 3715 at 12½*d.* 154 9½ 2|0)386|9 9½ *£193. 9s. 9½d.*	3215 at 1*s.* 1½*d.* *Ans.* £177. 9*s.* 10¾*d.*	3291 at 1*s.* 2½*d.* *Ans.* £195. 8*s.* 0¾*d.*
	2790 at 1*s.* 1¾*d.* *Ans.* £156. 18*s.* 9*d.*	9254 at 1*s.* 2¼*d.* *Ans.* £559. 1*s.* 11*d.*

7250 at 1*s.* 2¾*d.* *Ans.* £445. 11*s.* 5½*d.*	7524 at 1*s.* 6*d.* *Ans.* £564. 6*s.* 0*d.*	2571 at 1*s.* 9½*d.* *Ans.* £227. 12*s.* 9¾*d.*
7591 at 1*s.* 3*d.* *Ans.* £474. 8*s.* 9*d.*	7103 at 1*s.* 6½*d.* *Ans.* £540. 2*s.* 5¾*d.*	2104 at 1*s.* 9½*d.* *Ans.* £188. 9*s.* 8*d.*
6325 at 1*s.* 3½*d.* *Ans.* £401. 18*s.* 0½*d.*	3254 at 1*s.* 6½*d.* *Ans.* £250. 16*s.* 7*d.*	7506 at 1*s.* 9¾*d.* *Ans.* £680. 4*s.* 7½*d.*
5271 at 1*s.* 3½*d.* *Ans.* £340. 8*s.* 4½*d.*	7925 at 1*s.* 6¾*d.* *Ans.* £619. 2*s.* 9¾*d.*	1071 at 1*s.* 10*d.* *Ans.* £98. 3*s.* 0*d.*
3254 at 1*s.* 3¾*d.* *Ans.* £213. 10*s.* 10½*d.*	9271 at 1*s.* 7*d.* *Ans.* £733. 19*s.* 1*d.*	5200 at 1*s.* 10½*d.* *Ans.* £482. 1*s.* 8*d.*
2915 at 1*s.* 4*d.* *Ans.* £194. 6*s.* 8*d.*	7210 at 1*s.* 7½*d.* *Ans.* £578. 6*s.* 0½*d.*	2117 at 1*s.* 10½*d.* *Ans.* £198. 9*s.* 4½*d.*
3270 at 1*s.* 4½*d.* *Ans.* £221. 8*s.* 1½*d.*	2310 at 1*s.* 7½*d.* *Ans.* £187. 13*s.* 9*d.*	1007 at 1*s.* 10¾*d.* *Ans.* £95. 9*s.* 1¼*d.*
7059 at 1*s.* 4½*d.* *Ans.* £485. 6*s.* 1½*d.*	2504 at 1*s.* 7¾*d.* *Ans.* £206. 1*s.* 2*d.*	5000 at 1*s.* 11*d.* *Ans.* £479. 3*s.* 4*d.*
2750 at 1*s.* 4¾*d.* *Ans.* £191. 18*s.* 6¼*d.*	7152 at 1*s.* 8*d.* *Ans.* £596. 0*s.* 0*d.*	2105 at 1*s.* 11¼*d.* *Ans.* £203. 18*s.* 5¼*d.*
3725 at 1*s.* 5*d.* *Ans.* £263. 17*s.* 1*d.*	2905 at 1*s.* 8½*d.* *Ans.* £245. 2*s.* 2½*d.*	1006 at 1*s.* 11½*d.* *Ans.* £98. 10*s.* 1*d.*
7250 at 1*s.* 5½*d.* *Ans.* £521. 1*s.* 10½*d.*	7104 at 1*s.* 8½*d.* *Ans.* £606. 16*s.* 0*d.*	2705 at 1*s.* 11¾*d.* *Ans.* £267. 13*s.* 7½*d.*
2597 at 1*s.* 5½*d.* *Ans.* £189. 7*s.* 3½*d.*	1004 at 1*s.* 8¾*d.* *Ans.* £86. 16*s.* 1*d.*	5000 at 1*s.* 11½*d.* *Ans.* £489. 11*s.* 8*d.*
7210 at 1*s.* 5¾*d.* *Ans.* £533. 4*s.* 9½*d.*	2104 at 1*s.* 9*d.* *Ans.* £184. 2*s.* 0*d.*	4000 at £1*s.* 11¾*d.* *Ans.* £395. 16*s.* 8*d.*

RULE 4. When the price consists of any even number of shillings under 20, multiply the given quantity by half the price, doubling the first figure of the product for shillings, and the rest of the product will be pounds.

2750 at 2*s.* *Ans.* £275. 0*s.* 0*d.*	2710 at 6*s.* *Ans.* £813. 0*s.* 0*d.*	2102 at 10*s.* *Ans.* £1051. 0*s.* 0*d.*
3254 at 4*s.* *Ans. £650. 16s. 0d.*	1572 at 8*s.* *Ans.* £628. 16*s.* 0*d.*	2101 at 12*s.* Ans. £1260. 12*s.* 0*d.*

5271 at 14*s.* *Ans.* £3689. 14*s.* 0*d.*	1075 at 16*s.* *Ans.* £860. 0*s.* 0*d.*	NOTE. When the price is 10*s.*, take half of the quantity, and if any remains, it is 10*s.*
3123 at 16*s.* *Ans.* £2498. 8*s.* 0*d.*	1621 at 18*s.* *Ans.* £1458. 18*s.* 0*d.*	

RULE 5. When the price consists of odd shillings, multiply the given quantity by the price, and divide by 20, the product will be the answer.

2703 at 1*s.* *Ans.* £135. 3*s.* 0*d.*	2715 at 7*s.* *Ans.* £950. 5*s.* 0*d.*	2150 at 15*s.* *Ans.* £1612. 10*s.* 0*d.*
3270 at 3*s.* 3 2|0)981|0 £490. 10*s.* 0*d.*	3214 at 9*s.* *Ans.* £1446. 6*s.* 0*d.*	3142 at 17*s.* *Ans.* £2670. 14*s.* 0*d.*
	2710 at 11*s.* *Ans.* £1490. 10*s.* 0*d.*	2150 at 19*s.* *Ans.* £2042. 10*s.* 0*d.*
3271 at 5*s.* *Ans.* £817. 15*s.* 0*d.*	3179 at 13*s.* *Ans.* £2066. 7*s.* 0*d.*	7157 at 19*s.* *Ans.* £6799. 3*s.* 0*d.*

NOTE. When the price is 5*s.*, divide the quantity by 4, and the remainder, if any, is so many times 5*s.*

RULE 6. When the price is shillings and pence, and they the aliquot part of a pound, take that part of the quantity, and it will give the answer at once; but if they are not an aliquot part, then multiply the quantity by the shillings, and take parts for the rest, add them together and divide by 20.

s. *d.*			*d.*		
6 8	⅓	2710 at 6*s.* 8*d.* *Ans.* £903. 6*s.* 8*d.*	2	⅙	2710 at 3*s.* 2*d.* 3
		3150 at 3*s.* 4*d.* *Ans.* £525. 0*s.* 0*d.*			8130 451 8
		2715 at 2*s.* 6*d.* *Ans.* £339. 7*s.* 6*d.*		2|0	858|1 8 £429. 1*s.* 8*d.*
		7150 at 1*s.* 8*d.* *Ans.* £595. 16*s.* 8*d.*			7514 at 4*s.* 7*d.* *Ans.* £1721. 19*s.* 2*d.*
		3215 at 1*s.* 4*d.* *Ans.* £214. 6*s.* 8*d.*			2517 at 5*s.* 3*d.* *Ans.* £660. 14*s.* 3*d.*
		7211 at 1*s.* 3*d.* *Ans.* £450. 13*s.* 9*d.*			2547 at 7*s.* 3½*d.* Ans. £928. 11*s.* 10½*d.*

3271 at 5*s.* 9½*d.*
Ans. £943. 16*s.* 4¾*d.*

2103 at 15*s.* 4½*d.*
Ans. £1616. 13*s.* 7½*d.*

7152 at 17*s.* 6¾*d.*
Ans. £6280. 7*s.* 0*d.*

2510 at 14*s.* 7½*d.*
Ans. £1832. 16*s.* 5½*d.*

3715 at 9*s.* 4½*d.*
Ans. £1741. 8*s.* 1½*d.*

2572 at 13*s.* 7½*d.*
Ans. £1752. 3*s.* 6*d.*

7251 at 14*s.* 8½*d.*
Ans. £5324. 19*s.* 0¾*d.*

3210 at 15*s.* 7¾*d.*
Ans. £2511. 3*s.* 1½*d.*

2710 at 19*s.* 2½*d.*
Ans. £2602. 14*s.* 7*d.*

RULE 7. 1*st.* When the price is pounds and shillings, multiply the antity by the pounds, and proceed with the shillings, if they are even, in the 4th rule; if odd, take the aliquot parts, add them together, sum will be the answer.

2*ndly.* When pounds, shillings and pence, and the shillings and pence the aliquot parts of a pound, multiply the quantity by the pounds, d take parts for the rest.

3*rdly.* When the price is pounds, shillings, pence, and farthings, and shillings and pence not the aliquot parts of a pound, reduce the unds and shillings into shillings, multiply the quantity by the shillings, e parts for the rest, add them together, and divide by 20.

NOTE. When the given quantity is no more than three figures, prod as in Compound Multiplication.

4 | ½ | 7215 at £7. 4*s.* 0*d.*
7

50505
1443

£51948.

d.
6 | ⅛ | 2104 at £5. 3*s.* 0*d.*
5

10520
6 | ⅛ | 263
52 12

£10835. 12*s.*

2107 at £2. 8s. 0d.
Ans. £5056. 16s. 0d.

7156 at £5. 6*s.* 0*d.*
Ans. £37926. 16*s.* 0*d.*

6 | ½ | 2710 at £2. 3*s.* 7½*d.*
43

116530
1½ | ¼ | 1855
338 9

2|0 | 11822|3 9

£5911. 3*s.* 9*d.*

3215 at £1. 17*s.*
Ans. £5947. 15*s.* 0*d.*

2107 at £1. 13s.
Ans. £3478. 11s. 0d.

3215 at £4. 6*s.* 8*d.* *Ans.* £13931. 13*s.* 4*d.*	2157 at £2. 7 *Ans.* £5109. 7
2154 at £7. 1*s.* 3*d.* *Ans.* £15212. 12*s.* 6*d.*	142 at £1. 1£ *Ans.* £250. 2
2701 at £2. 3*s.* 4*d.* *Ans.* £5852. 3*s.* 4*d.*	95 at £15. 14 *Ans.* £1494.
2715 at £1. 17*s.* 2½*d.* *Ans.* £5051. 0*s.* 7½*d.*	37 at £1. 19 *Ans.* £73. 0:
2157 at £3. 15*s.* 2¼*d.* *Ans.* £8108. 19*s.* 5¼*d.*	2175 at £2. 1 *Ans.* £6022.
3210 at £1. 18*s.* 6¾*d.* *Ans.* £6189. 5*s.* 7½*d.*	2150 at £17. 1 *Ans.* £38283.

RULE 8. When the price and quantity given are of several nations, multiply the price by the integers, and take those par price answering to the parts of the integer for the rest.

At £3. 17*s.* 6*d.* per cwt., what is the value of 25 cwt. 2 qrs. tobacco?

qrs.					
2	½	£3	17	6	$5 \times 5 = 25$
				5	
		19	7	6	
				5	
lb.		96	17	6	
14	¼	1	18	9	
			9	8¼	
		£99	5	11¼	

2. At £1. 4*s.* 9*d.* per cwt., what comes 17 cwt. 1 qr. 17 lb. to? *Ans.* £21.

3. Sold 85 cwt. 1 qr. 10 lb. of cheese, at £1. 7*s.* 8*d.* per cw does it come to? *Ans.* £118.

4. Hops at £4. 5*s.* 8*d.* per cwt., what must be given for 72 c 18 lb? *Ans.* £31(

5. At £1. 1*s.* 4*d.* per cwt., what is the value of 27 cwt. 2 q of Malaga raisins? *Ans.* £29.

6. Bought 78 cwt. 3 qrs. 12 lb. of currants, at £2. 17*s.* 9*d.* what did I give for the whole? *Ans.* £227.

7. Sold 56 cwt. 1 qr. 17 lb. of sugar, at £2. 15s. 9d. the cw does it come to? Ans. £157

8. Tobacco at £3. 17*s.* 10*d.* the cwt., what is the worth of 97 cwt. 15 lb.? *Ans.* £378. 0*s.* 3*d.*

9. At £4. 14*s.* 6*d.* the cwt., what is the value of 37 cwt. 2 qrs. 13 lb. of double refined sugar? *Ans.* £177. 14*s.* 8½*d.*

10. Bought sugar at £3. 14*s.* 6*d.* the cwt., what did I give for 15 cwt. 1 qr. 10 lb. *Ans.* £57. 2*s.* 9*d.*

11. At £4. 15*s.* 4*d.* the cwt., the value of 172 cwt. 3 qrs. 12 lb. of tobacco is required? *Ans.* £823. 19*s.* 0½*d.*

12. Soap at £3. 11*s.* 6*d.* the cwt., what is the value of 53 cwt. 17 lb.? *Ans.* £190. 0*s.* 4*d.*

INTEREST.

INTEREST IS EITHER SIMPLE OR COMPOUND.

SIMPLE INTEREST

Is the *Profit* allowed in lending any sum of money, for a determined space of time.

The *Principal* is the money lent, for which Interest is to be received.

The *Rate per Cent.* is a certain sum agreed on between the borrower and the lender, to be paid for every £100, for the use of the Principal for 12 months.

The *Amount* is the principal and interest added together.

Interest is also applied to Commission, Brokerage, Purchasing of Stocks, and Insurance.

To find the Interest of any Sum of Money for a Year.

RULE. Multiply the principal by the rate per cent.; that product, divided by 100, will give the interest required.

For Several Years.

Multiply the interest of one year by the number of years given in the question, and the product will be the answer.

Examples.

1. What is the interest of £375 for a year, at 5 per cent. per annum?

5

18|75
20

15|00 *Ans.* £18. 15*s.*

2. What is the interest of £268 for one year, at 4 per cent. per annum? *Ans.* £10. 14*s.* 4¾*d.*

3. What is the interest of £945. 10*s.* for a year, at 4 per cent. per annum? *Ans.* £37. 16*s.* 4¾*d.*

4. What is the interest of £547. 15*s.*, at 5 per cent. per annum, for 3 years? *Ans.* £82. 3*s.* 3*d.*

5. What is the interest of £254. 17*s.* 6*d.* for 5 years, at 4 per cent. per annum? *Ans.* £50. 19*s.* 6*d.*

6. What is the interest of £556. 13*s.* 4*d.*, at 5 per cent. per annum, for 5 years? *Ans.* £139. 3*s.* 4*d.*

COMMISSION

Is an allowance from merchants to their factors or correspondents, in the buying or selling of goods, and is generally at a certain rate per cent., according to the custom of the country where the factor resides.

RULE. Multiply the principal by the rate per cent. as before; and for ¼, ½, or ¾, take the part or parts from the principal, which added to the product, and divided by 100, will give the answer.

7. What is the commission of £287. 10*s.*, at 3½ per cent.?

½ is ½,	287	10	
		3	
	862	10	for 3 per cent.
	143	15	,, ½ ,,
	10|06	5	,, 3½ ,,
		20	
	1|25		
		12	
	3|00		*Ans.* £10. 1*s.* 3*d.*

8. What must I allow my correspondent for disbursing on my account £529. 18*s.* 5*d.*, at 2¼ per cent. *Ans.* £11. 18*s.* 5½*d.*

9. My correspondent writes me word, that he has bought goods to the amount of £754. 16*s.*, on my account, what does his commission come to at 2½ per cent.? *Ans.* £18. 17*s.* 4¾*d.*

10. If I allow my factor 3¾ per cent. for commission, what may he demand on the laying out £876. 5*s.* 10*d.*? *Ans.* £32. 17*s.* 2½*d.*

PURCHASING OF STOCKS.

RULE. Multiply the sum to be purchased by the excess above 100; divide that product by 100, the quotient, added to the given sum, is *the purchase required.*

If under par (that is, under £100), multiply by the rate per cent.; that product, divided by 100, gives the purchase thereof.

11. What is the purchase of £575. 10*s.* Bank Stock, at 131¾ per cent.?

$6 \times 5 + 1 = 31$

575 10
6

3453
5

	17265	0	0	at 30	per c.*	
½ is ½	575	10	0	„ 1	„	
¼ „ ½	287	15	0	„ ½	„	
	143	17	6	„ ¼	„	
	182	72	2	6	31¾	

20

14|42
12

5|10

575 10 0
182 14 5

£758 4*s.* 5*d.* *Answer.*

12. What is the purchase of £254. 17*s.* Bank Annuities, at 97¼ per cent.?

$12 \times 8 + 1 = 97$

254 17
12

3058 4
8

	24465	12	0	at 96	per c.	
¼ is ¼	254	17	0	„ 1	„	
	63	14	3	„ ¼	„	
	247	84	3	3	97¼	

20

16|83
12

9|99
4

3|96

£247. 16*s.* 9¾*d.* *Answer.*
or more nearly £247. 16*s.* 10*d.*

13. At 110¼ per cent., what is the purchase of £2054. 16*s.* South Sea Stock? *Ans.* £2265. 8*s.* 4*d.*

14. At 104⅜ per cent. South Sea Annuities, what is the purchase of £1797. 14*s.*? *Ans.* £1876. 6*s.* 11¼*d.*

15. What is the purchase of £2750. 17*s.* South Sea Old Annuities, at 102⅜ per cent.? *Ans.* £2823. 1*s.* 2½*d.*

16. At 96¾ per cent., what is the purchase of £577. 19*s.* Bank Annuities? *Ans.* £559. 3*s.* 3¾*d.*

17. At 124⅝ per cent., what is the purchase of £758. 17*s.* 10*d.* India Stock? *Ans.* £945. 15*s.* 4¼*d.*

BROKERAGE

Is an allowance to brokers, for helping merchants or factors to persons to buy or sell them goods.

RULE. Divide the sum given by 100, and take parts of the quotient for the Rate per Cent.

* *These* are the per-centages after dividing by 100.

18. If I employ a broker to sell goods for me, to the value of £2575. 17*s.* 6*d.*, what is the brokerage, at 4*s.* per cent.?

£	*s.*	*d.*
25|75	17	6
20		
15|17		
12		
2|10		

	£	*s.*	*d.*
4*s.* ¼	25	15	2

Ans. £5. 3*s.* 0½*d.*

19. What is the brokerage of £796. 14*s.* 7*d.* at 6*s.* per cent.?
Ans. £2. 7*s.* 9½*d.*

20. When a broker sells goods to the amount of £7105. 5*s.* 10*d.*, what may he demand for brokerage, if he is allowed 5*s.* 6*d.* per cent.?
Ans. £19. 10*s.* 9½*d.*

21. If a broker is employed to buy a quantity of goods, to the value of £975. 6*s.* 4*d.*, what is the brokerage, at 6*s.* 6*d.* per cent.?
£3. 3*s.* 4½*d.*

When the time is ¼, ½, or ¾ of a year, besides a number of years given.

RULE. Take parts of the interest for one year, which add to the interest of the several years given, and it will give the answer.

22. What is the interest of £554. 10*s.* for 3 months at 4 per cent. per annum?

£.	*s.*
554	10
	4
22|18	0
20	
3|60	
12	
7|20	
22	3 7

m.		
3	¼	

Ans. £5. 10*s.* 10¾*d.*

23. What is the interest of £336. 15*s.* 6*d.* for 2¾ years, at 5 per cent. per annum?

£.	*s.*	*d.*
336	15	6
		5
16|83	17	6
20		
16|77		
12		
9|30		
4		
1|20		

½	½
¼	½

£.	*s.*	*d.*
16	16	9½
		2
33	13	6½
8	8	4½
4	4	2¼

Ans. £46 6 1¼

24. What is the interest of £325. 7*s.* 6*d.*, at 6 per cent. per annum, *for 3 years and a half?* *Ans.* £68. 6*s.* 6¼*d.*

25. What is the interest of £547. 2*s.* 4*d.* for 5 years and a half, at 4 r cent. per annum? *Ans.* £120. 7*s.* 3½*d.*

26. What is the interest of £257. 5*s.* 1*d.* at 4 per cent. for a year and ree quarters? *Ans.* £18. 0*s.* 1½*d.*

27. What is the interest of £479. 5*s.* 0*d.* for 5 years 1 quarter, at 5 r cent. per annum? *Ans.* £125. 16*s.* 0¾*d.*

When the Rate per Cent. is ¼, ½, or ¾, *more than the pounds given in e rate,* proceed as in commission, and it will give the answer for one ar; and, for several, proceed as in the last rule.

28. What is the interest of £175. 17*s.* 0*d.* for 2 years and 3 quarters, 4½ per cent. per annum?

½	¼	£.	s.	d.
		175	17	0
				4
		703	8	0
		87	18	6
		7|91	6	6
		20		
		18|26		
		12		
		3|18		

½	½	£.	s.	d.
		7	18	3
				2
		15	16	6
¼	½	3	19	1½
		1	19	6¾
		Ans. £21	15	2¼

29. What is the interest of £397. 9*s.* 5*d.* for 2 years and 1 quarter, 3½ per cent. per annum? *Ans.* £31. 6*s.*

30. What is the interest of £576. 2*s.* 7*d.* for 7 years and 1 quarter, 4½ per cent. per annum? *Ans.* £187. 19*s.* 1½*d.*

31. What is the interest of £279. 13*s.* 8*d.* at 5¼ per cent. per annum, r 3 years and a half? *Ans.* £51. 7*s.* 10*d.*

When the interest is required for any number of weeks.

RULE. As 52 weeks are to the weeks given, so is the interest of the ven sum for a year, to the interest required.

32. What is the interest of £379. 13*s.* 2*d.* for four weeks, at 4 per nt. *per annum?*

w.	w.	£.	s.	d.
As 52	: 4 ::	15	3	8¼
		20		
		303		
		12		
		3644		
		4		
		14578		
		4		
		52)58312(1121 farthings.		
		52		
		63	12)280¼	
		52	2|0)2|3	4¼
		111	£1	3 4¼ *Ans.*
		104		
		72		
		52		
		20		

£.	s.	d.
379	13	2
		4
15|18	12	8
	20	
3|72		
	12	
8|72		
	4	
2|88		

Or thus: multiply by number of weeks, and d the product by 4 and 13, t

4 × 13 = 52 13)15 3 8¼

Ans. £1 3 4¼

N.B. As it is 4 week merely divide by 13; as it is less to multiply by 4, and th divide by 4.

33. What is the interest of £259. 13*s.* 5*d.* for 20 weeks, at £ cent. per annum? *Ans.* £4. 19*s.* 1

34. What is the amount of £375. 6*s.* 1*d.* for 12 weeks, at 4½ per per annum? *Ans.* £379. 4*s.*

35. What is the amount of £256. 5*s.* 3*d.* for 25 weeks, at 2¾ per per annum? *Ans.* £259.

When the Interest is for any number of days.

RULE. Multiply the pence of the principal by the days and rate cent. for a dividend, cut off two figures on the right-hand, and d by 365, the quotient will be the answer in pence. Or,

As 365 days are to the days given, so is the interest of the given for a year, to the interest required.

36. What is the interest of £240 for 120 days, at 4 per cent. *annum?*

£.
240
f pence in £1, 240
57600
120
6912000 pence.
4
365)27648000(757 pence.
2555
2098
1825
2730
2555
175
4
700(1
365
335

2|0)6|3 1
£3. 3s. 1¼d.

Or thus:—

£. d. d. £. s. d.
240 as 365 : 120 :: 9 12 0
4 20
960 192
20 120
1200 365)23040(6|3 shills.
2190
£3. 3s. 1¼d.
1140
1095
45
12
540(1
365
175
4
700(1
365
335

What is the interest of £379. 5s. 4d. for 3 years and 75 days, at cent. per annum? *Ans.* £60. 15s. 8d.

At 5½ per cent. per annum, what is the interest of £985. 2s. 7d. years, 127 days? *Ans.* £289. 15s. 3d.

What is the interest of £2726. 1s. 4d., at 4½ per cent. per annum, years, 154 days? *Ans.* £419. 15s. 6½d.

When the amount, time, and rate per cent. are given, to find the principal.

LE. As the amount of £100 at the rate and time given : is to the nt given :: £100 : to the principal required.

What principal being put to interest will amount to £402. 10s. ears, at 3 per cent. per annum?

3 × 5 + 100 = £115 : £402 10s. :: £100
20 20
2300 8050
100
23|00)8050|00(£350 Ans.

41. What principal being put to interest for 9 years will amount to £734. 8*s.*, at 4 per cent. per annum? *Ans.* £540.

42. What principal being put to interest for 7 years, at 5 per cent. per annum, will amount to £334. 16*s.*? *Ans.* £248.

When the principal, rate per cent. and amount are given, to find the time.

RULE. As the interest of the principal for 1 year : is to the whole interest : : 1 year to the time required.

43. In what time will £350 amount to £402 10*s.*, at 3 per cent. per annum?

£.	Int. for 1 year.	Whole Int.
350	As 10 10	: 52 10 : 1 : : 5
3	20	20
10|50	210	21|0)105|0(5 years. *Ans.*
20		105
10|00		....

£402. 10*s.* 0*d.*
350 0 0
52 10 0

44. In what time will £540 amount to £734. 8*s.*, at 4 per cent. per annum? *Ans.* 9 *years.*

45. In what time will £248 amount to £334. 16*s.*, at 5 per cent. per annum? *Ans.* 7 *years.*

When the principal, amount, and time are given, to find the rate per cent. per annum.

RULE. As the principal : £100 : : interest for the whole time : interest of £100 for that time. Divide that interest by the number of years, and the quotient will be the rate per cent. per annum.

46. At what rate per cent. will £350 amount to £402. 10*s.* in 5 years time?

£.	s.	d.
402	10	0
350		
52	10	0

As 350 : 100 : : 52 10 : 15.

£. 52 20 1050 100

35\|0)10500\|0(300*s.* £15 ÷ 5 = 3 *per cent.*

47. At what rate per cent. will £248 amount to £334. 16*s.* in 7 years time? *Ans.* 5 *per cent.*

48. At what rate per cent. will £540 amount to £734. 8*s.* in 9 years time? *Ans.* 4 *per cent.*

COMPOUND INTEREST

Is that which arises both from the principal and interest; that is, when the interest on money becomes due, and not paid, the same interest is allowed on that unpaid interest, as was allowed on the principal.

Rule 1. Find the first year's interest, which add to the principal: then find the interest of that sum, which add as before, and so on for the number of years.

2. Subtract the given sum from the last amount, and it will give the compound interest required.

Examples.

1. What is the compound interest of £500 forborne 3 years, at 5 per cent. per annum?

£.	£.		£.	s.					
500	500		525	0					
5	25		26	5					
25|00	525	1st year.	551	5	2nd year.	£.	s.	d.	
	5			5		551	5	0	
	26|25		27|56	5		27	11	3	
	20			20		578	16	3	3rd year.
	5|00		11|25			500	0	0	prin. sub.
				12		78	16	3	= interest
			3|00						for 3 years.

2. What is the amount of £400 forborne 3½ years, at 6 per cent. per annum, compound interest? *Ans.* £490. 13*s.* 11½*d.*

3. What will £650 amount to in 5 years, at 5 per cent. per annum, compound interest? *Ans.* £829. 11*s.* 7½*d.*

4. What is the amount of £550. 10*s.* for 3 years and 6 months, at 6 per cent. per annum, compound interest? *Ans.* £675. 6*s.* 5*d.*

5. What is the compound interest of £764 for 4 years and 9 months, at 6 per cent. per annum? *Ans.* £243. 18*s.* 8½*d.*

6. What is the compound interest of £57. 10*s.* 6*d.* for 5 years, 7 months, and 15 days, at 4½ per cent. per annum? *Ans.* £18. 3*s.* 8½*d.*

7. What is the compound interest of £259. 10*s.* for 3 years, 9 months, and 10 days, at 4½ per cent. per annum? Ans. £48. 19*s.* 9*d.*

DISCOUNT

Is the abating so much money on a debt, to be received be due, as that money, if put to interest, would gain in the same t at the same rate: as £100 present money would discharge a £105 to be paid a year to come, discount being made at 5 per

RULE. As £100, with the interest for the time given : is to given : : so is the interest to the discount required.

Subtract the discount from the given sum, and the remainde the present worth.

Examples.

1. What is the discount and present worth of £487. 12*s.* for 6 at 6 per cent. per annum?

m.	£.		£.		*s.*		£.
6	½ 6	As 103	: 487		12	::	3
	3	20	20				
	100	2060	9752				
	103		3				
		206|0	)2925|6(	£14. 4*s.*	*discou*		
£487.	12*s.*		206				
14	4		865				
Ans. £473.	8*s.* *present worth.*		824				
			416				
			20				
			832|0(	4*s.*			
			824				
			8				

NOTE. Although the true discount is the sum here determ bankers always deduct for discount the whole *interest* on the counted. In the above example the banker's discount woul 12*s.* 6½*d.*

2. What is the present payment of £357. 10*s.*, which was be paid 9 months hence, at 5 per cent. per annum?

Ans. £344

3. What is the discount of £275. 10*s.* for 7 months, at 5 *per annum?* Ans. £7

4. Bought goods to the value of £109. 10*s.*, to be paid at 9 months, hat present money will discharge the same, if I am allowed 6 per cent. r annum discount? *Ans.* £104. 15*s.* 8¼*d.*

5. What is the present worth of £527. 9*s.* 1*d.*, payable 7 months nce, at 4½ per cent.? *Ans.* £514. 13*s.* 10¾*d.*

6. What is the discount of £85. 10*s.*, due September the 8th, this ing July the 4th, discount at 5 per cent. per annum? *Ans.* 15*s.* 3½*d.*

7. Sold goods for £875. 5*s.* 6*d.*, to be paid 5 months hence, what is e present worth at 4½ per cent.? *Ans.* £859. 3*s.* 4*d.*

8. What is the present worth of £500, payable in 10 months, at 5 r cent. per annum? *Ans.* £480.

9. How much ready money can I receive for a note of £75, due 15 nths hence, at 5 per cent.? *Ans.* £70. 11*s.* 9*d.*

10. What will be the present worth of £150, payable at three 4 nths; *i. e.*, one-third at 4 months, one-third at 8 months, and one-ird at 12 months, at 5 per cent. discount? *Ans.* £145. 3*s.* 8½*d.*

11. Sold goods to the value of £575. 10*s.*, to be paid at two 3 months, hat must be discounted for present payment, at 5 per cent.? *Ans.* £10. 11*s.* 4¾*d.*

12. What is the present worth of £500, at 4 per cent., £100 being to paid down, and the rest at two 6 months? *Ans.* £488. 7*s.* 8¼*d.*

EQUATION OF PAYMENTS.

When several sums are due at different times, it is often required to find nean time for paying the whole debt: to do which, this is the common Rule. Multiply each debt by its time, and divide the sum of the oducts by the whole debt: the quotient is accounted the mean time.

Examples.

1. A owes B £200, whereof £40 is to be paid at 3 months, £60 at 5 nths, and £100 at 10 months; at what time may the whole debt be id together, without prejudice to either?

£.		m.		
40	×	3	=	120
60	×	5	=	300
100	×	10	=	1000
		2|00)|14|20		
		7 months, $\frac{14}{20}$.		

2. B owes C £800, whereof £200 is to be paid at 3 months, £100 at months, £300 at 5 months, and £200 at 6 months; but they agreeing make but one payment of the whole, I demand what the time must ? (A fraction of a day is counted a whole day.) *Ans.* 4 *mo.*, 18 *d.*

3. I bought of K a quantity of goods to the value of £360, which as to have been paid as follows: £120 at 2 months, £200 at 4 months, id the *rest at 5 months;* but we afterwards agreed to have it paid *one mean time; the time is* demanded. Ans. 3 months, 13 days.

4. A merchant bought goods to the value of £500, to pay £100 at the end of 3 months, £150 at the end of 6 months, and £250 at the end of 12 months; but afterwards they agreed to discharge the debt at one payment; at what time was this payment made? *Ans.* 8 *months*, 12 *days*.

5. H is indebted to L a certain sum, which is to be paid at 6 different payments, that is, $\frac{1}{4}$ at 2 months, $\frac{1}{8}$ at 3 months, $\frac{1}{8}$ at 4 months, $\frac{1}{4}$ at 5 months, $\frac{1}{8}$ at 6 months, and the rest at 7 months; but they agree that the whole shall be paid at one equated time; what is that time? *Ans.* 4$\frac{1}{4}$ *months*.

6. A is indebted to B £120, whereof $\frac{1}{3}$ is to be paid at 3 months, $\frac{1}{4}$ at 6 months, and the rest at 9 months; what is the equated time of the whole payments? *Ans.* 5 *months*, 7 *days*.

BARTER

Is the exchanging one commodity for another, and informs the traders so to proportion their goods, that neither may sustain loss.

RULE. 1*st*. Find the value of that commodity whose quantity is given; then find what quantity of the other, at the rate proposed, you may have for the same money.

2*ndly*. When one has goods at a certain price, *ready money*, but in bartering advances it to something more, find what the other ought to rate his goods at, in proportion to that advance, and then proceed as before.

Examples.

1. What quantity of chocolate, at 4*s*. per lb., must be delivered in barter for 2 cwt. of tea, at 9*s*. per lb.?

2 cwt.
112
224
9
4|2016*s*. the value of the tea.
504 lb. of chocolate.

2. A and B barter; A has 20 cwt. of prunes, at 4*d*. per lb. ready money, but in barter will have 5*d*. per lb., and B has hops worth 32*s*. per cwt. ready money; what ought B to rate his hops at in barter, and what quantity must be given for the 20 cwt. of prunes?

112 as 4 : 5 :: 32*s*.
20 5
s. 40 12 — 2240 5 — 4)160
48|0)1120|0(23 cwt. 1 qr. 9$\frac{16}{48}$ lb. 40*s*. *Ans*.
96
160
144
16 = 1 qr. 9 lb. $\frac{16}{48}$.

3. How much tea, at 9*s*. per lb., can I have in barter for 4 cwt. 2 qrs. of *chocolate, at 4s*. per lb.? *Ans*. 2 *cwt*.

4. *Two merchants* barter; A has 20 cwt. of cheese, at 21*s*. 6*d*. per

cwt., B has 8 pieces of Irish cloth, at £3. 14*s.* per piece; I desire to know who must receive the difference, and how much?

Ans. B must receive of A £8. 2*s.*

5. A and B barter: A has 3½ lb. of pepper, at 13½*d.* per lb.; B has ginger at 15½*d.* per lb.; how much ginger must he deliver in barter for the pepper? *Ans.* 3 *lb.* 1$\frac{34}{61}$ *oz.*

6. How many dozen of candles, at 5*s.* 2*d.* per dozen, must be delivered in barter for 3 cwt. 2 qrs. 16 lb. of tallow, at 37*s.* 4*d.* per cwt.?

Ans. 26 *doz.* 3$\frac{54}{69}$ *lb.*

7. A has 608 yards of cloth, worth 14*s.* per yard, for which B gives him £125. 12*s.* in ready money, and 85 cwt. 2 qrs. 24 lb. of bees'-wax. The question is, what did B reckon his bees'-wax at per cwt.? *Ans.* £3. 10*s.*

8. A and B barter: A has 320 dozen of candles, at 4*s.* 6*d.* per dozen; for which B gives him £30 in money, and the rest in cotton, at 8*d.* per lb. I desire to know how much cotton B gave A besides the money? *Ans.* 11 *cwt.* 1 *qr.*

9. If B have cotton at 1*s.* 2*d.* per lb., how much must he give A for 114 lb. of tobacco, at 6*d.* per lb.? *Ans.* 48 $\frac{12}{14}$ *lb.*

10. C has nutmegs worth 7*s.* 6*d.* per lb. ready money, but in barter will have 8*s.* per lb., and D has leaf tobacco worth 9*d.* per lb. ready money; how much must D rate his tobacco at per lb. that his profit may be equivalent to C's? *Ans.* 9½*d.* $\frac{32}{55}$

PROFIT AND LOSS

Is a rule that discovers what is got or lost in the buying or selling of goods, and instructs us to rise or fall the price, so as to gain or lose so much per cent. or otherwise.

The questions in this rule are performed by the Rule of Three.

Examples.

1. If a yard of cloth is bought for 11*s.* and sold for 12*s.* 6*d.*, what is the gain per cent.?

As 11*s.* : 1*s.* 6*d.* :: £100

	12	20
	18	2000
12 6.		18
11 0		11)36000
1 6		12) 3272
		2|0) 27|2 8

Ans. £13. 12*s.* 8*d.* $\frac{8}{11}$

2. If 60 ells of Holland cost £18, what must 1 ell be sold for to gain 8 per cent.?

As 100 : 108 :: 18

	108
	12 × 5 = 60
1|00)19|44	
	20 12)19 8 9½
8|80	5)1 12 4¾
12	6 5¾
9|60	
4	
2|30	

Ans. 6*s.* 5¾*d.*

3. If 1 lb. of tobacco cost 16*d.* and is sold for 20*d.*, what is the gain per cent.? *Ans.* £25.

4. If a parcel of cloth be sold for £560, and at £12 per cent. gain, what was the prime cost? *Ans.* £500.

5. If a yard of cloth is bought for 13*s.* 4*d.*, and sold again for 16*s.*, what is the gain per cent.? *Ans.* £20.

6. If 112 lb. of iron cost 27*s.* 6*d.*, what must 1 cwt. be sold for to gain 15 per cent.? *Ans.* £1. 11*s.* 7½*d.*

7. If 375 yards of broad cloth be sold for £490 and 20 per cent. profit, what did it cost per yard? *Ans.* £1. 1*s.* 9½*d.*

8. Sold 1 cwt. of hops for £6. 15*s.*, at the rate of 25 per cent. profit, what would have been the gain per cent. if I had sold them for £8 per cwt.? *Ans.* £48. 2*s.* 11½*d.*

9. If 90 ells of cambric cost £60, how must I sell it per yard to gain £18 per cent.? *Ans.* 12*s.* 7*d.*

10. A plumber sold 10 fother of lead for £204. 15*s.* (the fother being 19½ cwt.) and gained after the rate of £12. 10*s.* per cent., what did it cost him per cwt.? *Ans.* 18*s.* 8*d.*

11. Bought 436 yards of cloth at the rate of 8*s.* 6*d.* per yard, and sold it for 10*s.* 4*d.* per yard; what was the gain of the whole? *Ans.* £39. 19*s.* 4*d.*

12. Paid £69 for one ton of steel, which is retailed at 6*d.* per lb.; what is the profit or loss by the sale of 14 tons? *Ans.* £182 *loss.*

13. Bought 124 yards of linen for £32, how should the same be retailed per yard to gain £15 per cent.? *Ans.* 5*s.* 11*d.* $\frac{88}{124}$.

14. Bought 249 yards of cloth, at 3*s.* 4*d.* per yard, retailed the same at 4*s.* 2*d.* per yard, what is the profit on the whole, and how much per cent.? *Ans.* £10. 7*s.* 6*d. profit, and* 25 *per cent.*

FELLOWSHIP

Is when two or more join their stock and trade together, so to determine each person's particular share of the gain or loss, in proportion to his principal in the joint stock.

By this rule a bankrupt's estate may be divided amongst his creditors; as also legacies may be adjusted when there is a deficiency of assets or effects.

FELLOWSHIP IS EITHER WITH OR WITHOUT TIME.

FELLOWSHIP WITHOUT TIME.

RULE. As the whole stock : each man's share in the stock :: the whole gain or loss : each man's share of the gain or loss.

PROOF. Add all the shares together, and the sum will be equal to *the given gain or loss*—but the surest way is, as the whole gain or

loss : the whole stock :: each man's share of the gain or loss : his share in stock.

Examples.

1. Two merchants trade together; A put into stock £20, and B £40, they gained £50, what is each person's share thereof?

		£.	s.	d.	
	20 + 40 = 60				
As 60 : 20 :: 50	as 60 : 40 :: 50	33	6	8	B's share.
20	40	16	13	4	A's ,,
6|0)100|0	6|0)200|0	50	0	0	
£16 13*s.* 4*d.*	£33 6*s.* 8*d.*				

2. Three merchants trade together, A, B, and C; A put in £20, B £30, and C £40; they gained £180; what is each man's part of the gain? *Ans.* A £40, B £60, C £80.

3. A, B, and C enter into partnership: A puts in £364, B £482, and C £500, and they gained £867; what is each man's share in proportion to his stock? *Ans.* A £234. 9*s.* 3½*d.*—*rem.* 70. B £310. 9*s.* 5*d.*—*rem.* 248. C £322. 1*s.* 3½*d.*—*rem.* 1028.

4. Four merchants, B, C, D, and E, made a stock; B put in £227, C £349, D £115, and E £439; in trading they gained £428, I demand each merchant's share of the gain?

Ans. B £85. 19*s.* 6¾*d.*—690. C £132. 3*s.* 9*d.*—120. D £43. 11*s.* 1¾*d.*—250. E £166. 5*s.* 6¼*d.*—70.

5. Three persons, D, E, and F, join in company; D's stock was £750, E's £460, and F's £500, and at the end of 12 months they gained £684; what is each man's particular share of the gain?

Ans. D £300, E £184, and F £200.

6. A merchant is indebted to B £275. 14*s.*, to C £304. 7*s.*, to D £152, and to E £104. 6*s.*, but upon his decease his estate is found to be worth but £675. 15*s.*; how must it be divided among his creditors?

Ans. B's £222. 15*s.* 2*d.*—6584, C's £245. 18*s.* 1½*d.*—15750, D's £122. 16*s.* 2¾*d.*—12227, and E's £84. 5*s.* 5*d.*—15620.

7. Four persons trading together in a joint-stock, of which A has ⅓, B ¼, C ⅛, and D ⅛, and at the end of six months they gain £100; what is each man's share of the said gain?

Ans. A £35. 1*s.* 9*d.*—48, B £26. 6*s.* 3¾*d.*—36, C £21. 1*s.* 0½*d*—120, and D £17. 10*s.* 10½*d.*—24.

8. Two persons purchased an estate of £1700 per annum, freehold, for £27200, when money was at 6 per cent. interest, and 4*s.* per pound land-tax, whereof D paid £15000, and E the rest; some time after, the interest of the money falling to 5 per cent. and 2*s.* per pound land-tax, they sell the said estate for 24 years' purchase; I desire to know each person's share? *Ans.* D £22500, E £18300.

9. D, E, *and* F *join their* stocks in trade: the amount of their stocks *is £647, and are in* proportion as 4, 6, and 8 are to one another, an

the amount of their gain is equal to D's stock; what is each man's stock and gain?

	£.	s.	d.			£.	s.	d.
D's stock.....	143	15	$6\frac{1}{2}\frac{2}{3}$	Gain.....		31	19	0
E's ,, ...	215	13	4	,, ...		47	18	6
F's ,, ...	287	1	$1\frac{1}{4}\frac{1}{3}$	,, ...		63	18	0

10. D, E, and F join stocks in trade: the amount of their stocks was £100, D's gain £8, E's £5, and F's £8; what was each man's stock?
Ans. D's stock £18. 15*s.*, E's £31. 5*s.*, and F's £50.

FELLOWSHIP WITH TIME.

RULE. As the sum of the products of each man's money and time: each man's product :: the whole gain or loss; his share of the gain or loss.

PROOF. As in Fellowship without Time.

Examples.

1. D and E enter into partnership; D puts in £40 for three months, and E £75 for four months, and they gained £70; what is each man's share of the gain?
Ans. D £20, E £50.

$40 \times 3 = 120$	as	420 : 120 :: 70	as	420 : 300 :: 70	
$75 \times 4 = 300$		120		300	
420		42|0)840|0(20		42|0)2100|0(50	

2. Three merchants join in company; D puts in stock £195. 14*s.* for 3 months, E £169. 18*s.* 3*d.* for 5 months, and F £59. 14*s.* 10*d.* for 11 months, they gained £364. 18*s.*; what is each man's part of the gain?
Ans. D's £102. 6*s.* 4*d.*—5008, E's £148. 1*s.* $1\frac{1}{2}d.$—482802, and F's £114. 10*s.* $6\frac{1}{2}d.$—14707.

3. Three merchants join in company for 18 months; D put in £500, and at 5 months' end took out £200; at 10 months' end put in £300, and at the end of 14 months takes out £130. E puts in £400, and at the end of 3 months £270 more; at 9 months he takes out £140, but puts in £100 at the end of 12 months, and withdraws £99 at the end of 15 months. F puts in £900, and at 6 months took out £200; at the end of 11 months put in £500, but takes out that and £100 more at the end of 13 months. They gained £200; I desire to know each man's share of the gain?
Ans. D £50. 7*s.* 6*d.*—21720, E £62. 12*s.* $5\frac{1}{2}d.$—29859, and F £87. 0*s.* $0\frac{1}{2}d.$—14167.

4. D, E, and F hold a piece of ground in common, for which they are to pay £36. 10*s.* 6*d.* D puts in 23 oxen 27 days; E 21 oxen 35 days; and F 16 oxen 23 days. What is each man to pay of the said rent?
Ans. D £13. 3*s.* $1\frac{1}{2}$d.—624, E £15. 11*s.* 5d.—1888, and F £7. 15*s.* 11d.—1136.

ALLIGATION.

ALLIGATION IS EITHER MEDIAL OR ALTERNATE.

ALLIGATION MEDIAL

Is when the price and quantities of several simples are given to be mixed, to find the mean price of that mixture.

Rule. As the whole composition is to any part of it, so is the value of the whole to the value of that part.

Proof. Find the value of the whole mixture at the mean rate, and if it agrees with the total value of the several quantities at their respective prices, the work is right.

Examples.

1. A farmer mixed 20 bushels of wheat, at 5*s.* per bushel, and 36 bushels of rye, at 3*s.* per bushel, with 40 bushels of barley, at 2*s.* per bushel. I desire to know the worth of a bushel of this mixture.

		bush. bush.
20 × 5 =	100	as 96 : 1 :: 288 : 3
36 × 3 =	108	
40 × 2 =	80	
96	288	*Ans.* 3*s.*

2. A vintner mingles 15 gallons of Canary, at 8*s.* per gallon, with 20 gallons at 7*s.* 4*d.* per gallon, 10 gallons of sherry at 6*s.* 8*d.* per gallon, and 24 gallons of white wine at 4*s.* per gallon; what is the worth of a gallon of this mixture? *Ans.* 6*s.* $2\frac{1}{2}$*d.* $\frac{48}{69}$.

3. A grocer mingled 4 cwt. of sugar at 56*s.* per cwt., 7 cwt. at 43*s.* per cwt., and 5 cwt. at 37*s.* per cwt.; I demand the price of 2 cwt. of this mixture? *Ans.* £4. 8*s.* 9*d.*

4. A maltster mingles 30 quarters of brown malt, at 28*s.* per quarter, with 46 quarters of pale, at 30*s.* per quarter, and 24 quarters of high-dried ditto, at 25*s.* per quarter. What is the value of 8 bushels of this mixture? *Ans.* £1. 8*s.* $2\frac{1}{4}$*d.* $\frac{6}{10}$.

5. If I mix 27 bushels of wheat, at 5*s.* 6*d.* per bushel, with the same quantity of rye, at 4*s.* per bushel, and 14 bushels of barley, at 2*s.* 8*d.* per bushel, what is the worth of a bushel of this mixture?

Ans. 4*s.* $3\frac{3}{4}$*d.* $\frac{3}{10}$.

6. *A grocer mingled* 3 cwt. of sugar, at 56*s.* per cwt., 6 cwt. at £1. 17*s.* 4*d.* *per cwt.*, and 3 cwt. at £3. 14*s.* 8*d.* per cwt. What is 1 cwt. of this mixture worth?

Ans. £2. 11*s.* 4

7. A mealman has flour of several sorts, and would mix 3 bushels at 3*s.* 5*d.* per bushel, 4 bushels at 5*s.* 6*d.* per bushel, and 5 bushels at 4*s.* 8*d.* per bushel. What is the worth of a bushel of this mixture?

Ans. 4*s.* 7$\frac{1}{2}$*d.* $\frac{4}{13}$.

8. A vintner mixes 20 gallons of port, at 5*s.* 4*d.* per gallon, with 12 gallons of white wine, at 5*s.* per gallon, 30 gallons of Lisbon, at 6*s.* per gallon, and 20 gallons of mountain, at 4*s.* 6*d.* per gallon. What is a gallon of this mixture worth?

Ans. 5*s.* 3$\frac{3}{4}$*d.* $\frac{5}{8}$.

9. A farmer mingled 20 bushels of wheat, at 5*s.* per bushel, and 36 bushels of rye, at 3*s.* per bushel, with 40 bushels of barley, at 2*s.* per bushel. I desire to know the worth of a bushel of this mixture?

Ans. 3*s.*

10. A person mixing a quantity of oats, at 2*s.* 6*d.* per bushel, with the like quantity of beans, at 4*s.* 6*d.* per bushel, would be glad to know the price of one bushel of that mixture.

Ans. 3*s.* 6*d.*

11. A refiner having 12 lb. of silver bullion, of 6 oz. fine, would melt it with 8 lb. of 7 oz. fine, and 10 lb. of 8 oz. fine; required the fineness of 1 lb. of that mixture?

Ans. 6 *oz.* 18 *dwt.* 16 *gr.*

12. If with 40 bushels of corn, at 4*s.* per bushel, there are mixed 10 bushels at 6*s.* per bushel, 30 bushels at 5*s.* per bushel, and 20 bushels at 3*s.* per bushel, what will 10 bushels of that mixture be worth?

Ans. £2. 3*s.*

13. A tobacconist would mix 50 lb. of tobacco, at 11*d.* per lb., with 30 lb. at 14*d.* per lb., 25 lb. at 22*d.* per lb., and 37 lb. at 2*s.* per lb. What will 1 lb. of this mixture be worth?

Ans. 16$\frac{3}{4}$*d.* $\frac{11}{148}$.

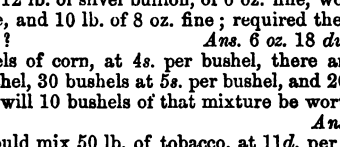
ALLIGATION ALTERNATE

Is when the price of several things are given, to find such quantities of them as will make a mixture, that may bear a proposed price.

In ordering the rates and given price, observe—

1. Place them one under the other, and the proposed price, or mean rate, at the left hand of them, thus:

(*See Ex.* 1, *next page.*)



4. Link the several rates together by two and two, always observing to join a greater and a less than the mean.

3. Against each extreme, place the difference of the mean and its yoke-fellow.

When the prices of the several samples and the mean rate are given without any quantity, to find how much of each simple is required to compose the mixture.

RULE. Take the difference between each price and the mean rate, and set them alternately, they will be the answer required.

PROOF. By Alligation Medial.

Examples.

1. A vintner would mix four sorts of wines together, of 18*d.*, 20*d.*, 24*d.*, and 28*d.* per quart; what quantity of each must he take to sell the mixture at 22*d.* per quart?

	Answer.		*Proof.*
18	2 of 18*d.*	=	36*d.*
22 20	6 of 20*d.*	=	120
24	4 of 24*d.*	=	96
28	2 of 28*d.*	=	56
14			)308
			22*d.*

	Or thus:		*Proof.*
18	6 of 18*d.*	=	108*d.*
22 20	2 of 20*d.*	=	40
24	2 of 24*d.*	=	48
28	4 of 28*d.*	=	112
14			)308
			22*d.*

NOTE. Questions in this rule admit of a great variety of answers, according to the manner of linking them.

2. A grocer would mix sugar at 4*d.*, 6*d.*, and 10*d.* per lb., so as to sell the compound for 8*d.* per lb.; what quantity of each must he take?

Ans. 2 *lb. at* 4*d.*, 2 *lb. at* 6*d.*, *and* 6 *lb. at* 10*d.*

3. I desire to know how much tea at 16*s.*, 14*s.*, 9*s.*, and 8*s.* per lb., will compose a mixture worth 10*s.* per lb.?

Ans. 1 *lb. at* 16*s.*, 2 *lb. at* 14*s.*, 6 *lb. at* 9*s.*, *and* 4 *lb. at* 8*s.*

4. A farmer would mix as much barley at 3*s.* 6*d.* per bushel, rye at 4*s.* per bushel, and oats at 2*s.* per bushel, as to make a mixture worth 2*s.* 6*d.* per bushel. How much is that of each sort?

Ans. 6 *of barley*, 6 *of rye, and* 30 *of oats.*

5. A grocer would mix raisins of the sun at 7*d.* per lb., with Malaga's at 6*d.* and Smyrna's at 4*d.* per lb. I desire to know what quantity of each sort he must take to sell them at 5*d.* per lb.?

Ans. 1 *lb. of raisins of the sun*, 1 *lb. of Malaga's, and* 3 *lb. of Smyrna's.*

6. A tobacconist would mix tobacco of 2*s.*, 1*s.* 6*d.*, and 1*s.* 3*d.* per lb., so as the compound may bear a price of 1*s.* 8*d.* per lb. What quantity of each sort must he take?

Ans. 7 *lb. at* 2*s.*, 4 *lb. at* 1*s.* 6*d.*, *and* 4 *lb. at* 1*s.* 3*d.*

ALLIGATION PARTIAL.

IS when the prices of all the simples, the quantity of only one of them, and the mean rate are given, to find the several quantities of the rest in proportion to that given.

RULE. Take the difference between each price, and the mean rate, as before. Then,

As the difference of that simple, whose quantity is given : the rest of the differences severally :: the quantity given to the several quantities *required.*

Examples.

1. A tobacconist being determined to mix 20 lb. of tobacco per lb., with others at 16*d.* per lb., 18*d.* per lb., and 22*d.* per many pounds of each sort must he take to make 1 pound of ture worth 17*d.*?

		Answer.	*Proof.*	
15	5	20 lb. at 15*d.*	= 300*d.*	As 5 : 1 ::
17 16	1	4 lb. at 16*d.*	= 64*d.*	As 5 : 1 ::
18	1	4 lb. at 18*d.*	= 72*d.*	As 5 : 2 ::
22	2	8 lb. at 22*d.*	= 176*d.*	
		36 lb. : 1 lb. ::	612*d.* : 17*d.*	

This last proportion shows that the whole mixture, consisting and of which the value is 612*d.*, is worth 17*d.* per lb.

2. A farmer would mix 20 bushels of wheat, at 60*d.* per bus rye at 36*d.*, barley at 24*d.*, and oats at 18*d.* per bushel; h must he take of each sort to make the composition worth bushel? *Ans.* 20 *bushels of wheat*, 35 *bushels of rye*, 7 *of barley, and* 10 *bushels*

3. A person is desirous of mixing wheat at 4*s.* per bushel, r per bushel, and barley at 2*s.* per bushel, with 12 bushels o 18*d.* per bushel; I would be glad to know how many bushels sort he must take to make the composition worth 3*s.* 6*d.* per *Ans.* 96 *bushels of wheat*, 12 *bushels of rye*, 12 *of barley, and* 1

4. A distiller would mix 40 gallons of French brandy, at gallon, with English at 7*s.* and spirits at 4*s.* per gallon; what of each sort must he take to afford it for 8*s.* per gallon? *Ans.* 40 *gallons French*, 32 *English, and* 3

5. A grocer would mix teas of 12*s.*, 10*s.*, and 6*s.*, with 20 per lb.; how much of each sort must he take to make the con worth 8*s.* per lb.? *Ans.* 20 *lb. at* 4*s.*, 10 *lb. at* 6*s.*, 10 *lb. at* 10*s., and* 20 *l*

6. A wine merchant is desirous of mixing 18 gallons of C 6*s.* 9*d.* per gallon, with Malaga at 7*s.* 6*d.* per gallon, sherry gallon, and white wine at 4*s.* 3*d.* per gallon; how much of must he take that the mixture may be sold for 6*s.* per gallon? *Ans.* 18 *gallons of Canary*, 31½ *of Malaga*, 13½ *and* 27 *of wi*

ALLIGATION TOTAL

Is when the price of each simple, the quantity to be com and the mean rate are given, to find how much of each sort w that quantity.

Rule. Take the difference between each price, and the m *as before; then,*

As the sum of the differences : each particular difference :: the quantity given : the quantity required.

Examples.

1. A grocer has four sorts of sugar, viz. at 12*d.*, 10*d.*, 6*d.*, and 4*d.* per lb., and would make a composition of 144 lb. worth 8*d.* per lb. I desire to know what quantity of each he must take.

		Answer.			*Proof.*			lb.	lb.
8	12	4	48	at 12*d.*	576	As 12 : 4 :: 144 : 48			
	10	2	24	„ 10*d.*	240	As 12 : 2 :: 144 : 24			
	6	2	24	„ 6*d.*	144				
	9	4	48	„ 4*d.*	142				
		12	144		)1152(8*d.*				

2. A grocer having four sorts of tea, at 5*s.*, 6*s.*, 8*s.*, and 9*s.* per lb., would have a composition of 87 lb. worth 7*s.* per lb. What quantity must there be of each?

Ans. 14½ *lb. of* 5*s.*, 29 *lb. of* 6*s.*, 29 *lb. of* 8*s.*, *and* 14½ *lb. of* 9*s.*

3. A vintner having four sorts of wine, viz. white wine, at 4*s.* per gallon; Flemish, at 6*s.* per gallon; Malaga, at 8*s.* per gallon; and Canary, at 10*s.* per gallon, would make a mixture of 60 gallons, to be worth 5*s.* per gallon. What quantity of each must he take?

Ans. 45 *gallons of white wine, 5 gallons of Flemish, 5 gallons of Malaga, and 5 gallons of Canary.*

4. A grocer having four sorts of currants, of 11*d.*, 9*d.*, 6*d.*, and 4*d.* per lb., is desirous of making a composition of 240 lb., worth 8*d.* per lb. How much of each must he take?

Ans. 96 *lb. at* 11*d.*, 48 *lb. at* 9*d.*, 24 *lb. at* 6*d.*, *and* 72 *lb. at* 4*d.*

5. A silversmith having four sorts of gold, viz. of 24 carats fine, of 22, 20, and 15 carats fine; would make as much of each sort together, as to have 42 oz. of 17 carats fine. How much must he take of each?

Ans. 4 *of* 24, 4 *of* 22, 4 *of* 20, *and* 30 *of* 15 *carats fine.*

6. A druggist having some drugs of 8*s.*, 5*s.*, and 4*s.* per lb., made them into two parcels: one of 28 lb. at 6*s.* per lb., the other of 42 lb. at 7*s.* per lb. How much of each sort did he take for each parcel?

Ans. 12 *lb. of* 8*s.*	*Ans.* 30 *lb. of* 8*s.*
8 *lb. of* 5*s.*	6 *lb. of* 5*s.*
8 *lb. of* 4*s.*	6 *lb. of* 4*s.*
28 *lb. at* 6*s. per lb.*	42 *lb. at* 7*s. per lb.*

POSITION, OR THE RULE OF FALSE,

Is a rule that, by false or supposed numbers, taken at pleasure, discovers the true *ones* required. It is divided into two parts; SINGLE *and* DOUBLE.

SINGLE POSITION

Is when, by using one supposed number, and working with it
true one, you find the real number required, by the following

RULE. As the total of the errors : the true total :: the sup
number : the true one required.

PROOF. Add the several parts of the sum together, and, if it
with the amount stated, the work is right.

Examples.

1. A schoolmaster being asked how many scholars he had, said,
had as many, half as many, and one quarter as many more, I
have 88." How many had he? *A*

Suppose he had	40	As 110 : 88 :: 40 : 32		
As many	40	40	32	*as many*
½ *as many*	20	———	16	½ *as ma*
¼ *as many*	10	11|0)352|0(32	8	¼ *as ma*
	110		88	*proof.*

2. A person having about him a certain number of Portugal
said, if the third, fourth, and sixth of them were added together
would make 54. I desire to know how many he had? *A*

3. A gentleman bought a chaise, horse and harness, for £6
horse came to twice the price of the harness, and the chaise to tw
price of the horse and harness. What did he give for each?
Ans. Horse £13. 6*s.* 8*d.*, *harness* £6. 13*s.* 4*d.*, *chai*

4. A, B, and C being determined to buy a quantity of goods,
would cost them £120, agreed amongst themselves that B should
third part more than A, and C a fourth part more than B. I de
know what each man must pay? *Ans. A* £30, *B* £40,

5. A man overtaking a maid driving a flock of geese, said t
"How do you do, sweetheart? where are you going with th
geese?" "No, sir," said she, "I have not 30; but if I
many more, half as many more, and 5 geese besides, I should ha
How many had she? *A*

6. A person delivered to another a sum of money, unkno
receive interest for the same at 6 per cent. per annum, simple in
and at the end of 10 years received for principal and interest
What was the sum lent? *Ans.* £1

DOUBLE POSITION

In double position you make use of two supposed numbers,
both prove false (as generally happens), they are with their er
bo thus ordered.

Rule 1. Place each error against its respective supposition.

2. Multiply them crosswise.

3. If the errors are alike, *i.e.*, both greater, or both less than the given number, take their difference for a divisor, and the difference of the products for a dividend. But if unlike, take their sum for a divisor, and the sum of the products for a dividend, the quotient will be the answer.

Examples.

1. A, B, and C would divide £200 between them, so that B may have £6 more than A, and C £8 more than B. How much must each have?

Suppose A had 40
then B had 46
and C..... 54

140 too little by 60
sup. errors.
40 × 60
50 × 30
3000 1200
1200

S|0)180|0

60 *Ans. for* A.

Then suppose A had 50
then B must have 56
and C 64

170 too little by 30.

60
30
—
30 *divisor.*

60 A
66 B
74 C
200 *proof.*

2. A man had two silver cups, of unequal weight, having one cover to both of 5 oz.; now if the cover is put on the lesser cup, it will double the weight of the greater cup; and set on the greater cup, it will be thrice as heavy as the lesser cup. What is the weight of each cup?

Ans. 3 *ounces lesser*, 4 *greater.*

3. A, B, and C, playing at hazard together, the money staked was 196 guineas; but disagreeing, each seized as many as he could: A got a certain quantity; B as many as A, and 16 more; and C the 6th part of both their sums. How many had each?

Ans. A 76, B 92, and C 28.

4. A gentleman bought a house with a garden, and a horse in the stable, for £500; now he paid 4 times the price of the horse for the garden, and 5 times the price of the garden for the house. What was the value of the house, garden, and horse, separately?

Ans. Horse £20, *garden* £80, *house* £400.

5. Three persons discoursed concerning their ages; says H, I am 30 years of age; says K I am as old as H, and ¼ of L; and says L, I am as old as you both. What was the age of each person?

Ans. H 30, K 50, and L 80.

6. D, E, and F, playing at cards, staked 324 crowns; but disputing about the tricks, each man took as many as he could: D got a certain number; E as many as D, and 15 more; and F got a fifth part of both their sums added together. How many did each get?

Ans. D 127½, E 142½, and F 54.

7. A, stealing apples, was taken by B, and to appease him gave him half of what he had, and B gives him back 10; going farther he meets C, who took from him half of what he had left, and gives him back 4; after that, meeting with D, he gives him half of what he had, and he returns him back 1; at last getting safe away, he finds he had 13 left. How many had he at first? *Ans.* 60.

8. A gentleman going into a garden, meets with some ladies, and says to them, "Good morning to you 10 fair maids." "Sir, you mistake," answered one of them, "we are not ten; but if we were twice as many more as we are, we should be as many above 10 as we are now under." How many were they? *Ans.* 5.

EXCHANGE

Is the receiving money in one country for the same value paid in another.

The par of exchange is always fixed and certain, it being the intrinsic value of foreign money compared with sterling; but the course of exchange rises and falls upon various occasions.

I. FRANCE.

They keep their accounts at Paris, Lyons, and Rouen, in livres, sols, and deniers, and exchange by the crown = 4*s.* 6*d.* at par.

NOTE. 12 deniers make ..... 1 sol.
20 sols ..... 1 livre.
3 livres ..... 1 crown.

To change French into Sterling.

RULE. As 1 crown : the given rate :: the French sum : the sterling required.

To change Sterling into French.

RULE. As the rate of exchange : 1 crown :: the sterling sum : the *French required.*

Examples.

1. How many crowns must be paid at Paris to receive in London £180 exchange, at 4*s.* 6*d.* per crown?

	d.		*c.*		£.	
As	54	:	1	::	180	:
No. of pence in £1 ..			240			

crowns.

54)43200(800
432

....

2. A merchant at Paris remits to his correspondent in London 800 crowns, at 4*s.* 6*d.* each; what is the value in sterling?

cr.		*d.*		*cr.*	
1	:	54	::	800	:
				54	

12)43200

2|0)360|0

£180

3. How much sterling must be paid in London, to receive in Paris 758 crowns, exchange at 56*d.* per crown? *Ans.* £176. 17*s.* 4*d.*

4. A merchant in London remits £176. 17*s.* 4*d.* to his correspondent at Paris; what is the value of French crowns, at 56*d.* per crown?

Ans. 758.

5. Change 725 crowns, 17 sols, 7 deniers, at 54½*d.* per crown, into sterling; what is the sum? *Ans.* £164. 14*s.* 0½*d.*

6. Change £164. 14*s.* 0½*d.* sterling into French crowns, exchange at 54½*d.* per crown? *Ans.* 725 *crowns*, 17 *sols*, 7 *deniers*.

II. SPAIN.

They keep their accounts at Madrid, Cadiz, and Seville, in dollars, rials, and maravedies, and exchange by the piece of eight = 4*s.* 6*d.* at par.

NOTE. 34 maravedies make ..... 1 rial.
8 rials ..... 1 piastre, or piece of eight.
10 rials ..... 1 dollar.

RULE. As with France.

Examples.

7. A merchant at Cadiz remits to London 2547 pieces of eight, at 56*d.* per piece, how much sterling is the sum? *Ans.* £594. 6*s.*

8. How many pieces of eight, at 56*d.* each, will answer a bill of £594. 6*s.* sterling? *Ans.* 2547.

9. If I pay a bill here of £2500, what Spanish money may I draw my bill for at Madrid, exchange at 57½*d.* per piece of eight?

Ans. 10434 *pieces of eight*, 6 rials, $8\frac{20}{25}$ mar.

III. ITALY.

They keep their accounts at Genoa and Leghorn in livres, sols, and deniers, and exchange by the piece of eight, or dollar = 4*s*. 6*d*. at par.

NOTE. 12 deniers make..... 1 sol.
20 sols..... 1 livre.
5 livres..... 1 piece of eight at Genoa.
6 livres..... 1 piece of eight at Leghorn.

N.B. The exchange at Florence is by ducatoons; the exchange at Venice by ducats.

NOTE. 6 solidi make..... 1 gross.
24 grosses..... 1 ducat.

RULE. The same as before.

10. How much sterling money may a person receive in London, if he pays in Genoa 976 dollars, at 53*d*. per dollar? *Ans*. £215. 10*s*. 8*d*.

11. A merchant remitted £215. 10*s*. 8*d*. sterling to Leghorn, how many dollars will he receive there, the exchange being at 53*d*. per dollar? *Ans*. 976.

12. A factor having sold goods at Florence for 250 ducatoons, at 54*d*. each, what is the value in pounds sterling? *Ans*. £56. 5*s*.

13. A bill of £56. 5*s*. is remitted to Florence, to be paid in ducatoons, at 54*d*. each, how many will be received? *Ans*. 250.

14. If 275 ducats, at 4*s*. 5*d*. each, be remitted from Venice to London, what is the value in pounds sterling? *Ans*. £60. 14*s*. 7*d*.

15. A gentleman travelling, would exchange £60. 14*s*. 7*d*. sterling for Venice ducats, at 4*s*. 5*d*. each, how many must he receive?
Ans. 275.

IV. PORTUGAL.

They keep their accounts in Oporto and Lisbon, in reas, and exchange on the milrea = 6*s*. 8½*d*. at par.

NOTE. 1000 reas make..... 1 milrea.

RULE. The same as with France.

Examples.

16. A gentleman being desirous to remit to his correspondent in London 2750 milreas, exchange at 6*s*. 5*d*. per milrea, how much sterling will he be the creditor for in London? *Ans*. £882. 5*s*. 10*d*.

17. If a bill be drawn from London of £882. 5*s*. 10*d*. sterling, how many milreas, at 6*s*. 5*d*. each, is equal in value to the said sum?
Ans. 2750

18. A merchant at Oporto remits to London 4366 milreas, and 183 reas, at 5*s.* 5$\frac{3}{8}$*d.* exchange per milrea, how much sterling must be paid in London for this remittance? *Ans.* £1193. 17*s.* 6$\frac{3}{4}$*d.*

19. If I pay a bill at London of £1193. 17*s.* 6$\frac{3}{4}$*d.*, what must I draw for on my correspondent at Lisbon, exchange at 5*s.* 5$\frac{3}{8}$*d.* per milrea? *Ans.* 4366 *milreas,* 183 *reas.*

V. HOLLAND, FLANDERS, AND GERMANY.

They keep their accounts at Antwerp, Amsterdam, Brussels, Rotterdam, and Hamburgh; some in pounds, shillings, and pence, as in England; others in guilders, stivers, and pennings; and exchange with us on our pound, at 33*s.* 4*d.* Flemish, as par.

NOTE. 8 pennings make ..... 1 groat.
2 groats, or 16 pennings..... 1 stiver.
20 stivers ..... 1 guilder or florin.

ALSO,

12 groats, or 6 stivers, make ..... 1 schelling.
20 schellings, or 6 guilders ..... 1 pound.

To change Flemish into Sterling.

RULE. As the given rate : is to 1 pound : : so is the Flemish sum : to the sterling required.

To change Sterling into Flemish.

RULE. As £1 sterling : is to the given rate : : so is the sterling given : to the Flemish sought.

Examples.

20. Remitted from London to Amsterdam, a bill of £754. 10*s.* sterling, how many pounds Flemish is the sum, the exchange at 33*s.* 6*d.* Flemish per pound sterling? *Ans.* £1263. 15*s.* 9*d. Flemish.*

21. A merchant at Rotterdam remits £1263. 15*s.* 9*d.* Flemish to be paid in London, how much sterling money must he draw for, the exchange being at 33*s.* 6*d.* Flemish per pound sterling? *Ans.* £754. 10*s.*

22. If I pay in London £852. 12*s.* 6*d.* sterling, how many guilders must I draw for at Amsterdam, exchange at 34 schel. 4$\frac{1}{2}$ groats Flemish per pound sterling? *Ans.* 8792 *guild.*, 13 *stiv.*, 14$\frac{1}{2}$ *pennings.*

23. What must I draw for at London, if I pay at Amsterdam 8792 guild. 13 stiv. 14$\frac{1}{2}$ pennings, exchange at 34 schel. 4$\frac{1}{2}$ groats per pound *sterling?* Ans. £852. 12s. 6d.

To convert Bank Money into Current, and the contrary.

NOTE. The Bank Money is worth more than the Current. The difference between the one and the other is called *agio*, and is generally from 3 to 6 per cent. in favour of the Bank.

To change Bank into Current money.

RULE. As 100 guilders Bank : is to 100 with the agio added :: so is the Bank given : to the current required.

To change Current money into Bank.

RULE. As 100, with the agio added : is to 100 Bank :: so is the Current money given : to the Bank required.

24. Change 794 guilders, 15 stivers, Current money, into Bank florins, agio $4\frac{3}{8}$ per cent. *Ans.* 761 *guilders*, 8 *stivers*, 11 *pennings*.

25. Change 761 guilders, 9 stivers, Bank into Current money, agio $4\frac{3}{8}$ per cent. *Ans.* 794 *guilders*, 15 *stivers*, 4 *pennings*.

COMPARISON OF WEIGHTS AND MEASURES.

Examples.

1. If 50 Dutch pence be worth 65 French pence, how many Dutch pence are equal to 350 French pence? *Ans.* $269\frac{11}{13}$.

2. If 12 yards at London make 8 ells at Paris, how many ells at Paris will make 64 yards at London? *Ans.* $42\frac{8}{12}$.

3. If 30 lb. at London make 28 lb. at Amsterdam, how many lb. at London will be equal to 350 lb. at Amsterdam? *Ans.* 375.

4. If 95 lb. Flemish make 100 lb. English, how many lb. English are equal to 275 lb. Flemish? *Ans.* $289\frac{45}{95}$.

CONJOINED PROPORTION,

ALSO CALLED THE CHAIN RULE,

Is used when the coin, weight, or measures of several countries are compared in the same question; and it is required to find how many of the first sort of coin, weight, or measure mentioned in the question are equal to a given quantity of the last.

RULE. Place the numbers alternately, beginning at the left-hand, and let the last number stand on the left-hand; then multiply the first row continually for a dividend, and the second for a divisor.

PROOF. By as many Single Rules of Three as the question requires.

Examples.

1. If 20 lb. at London make 23 lb. at Antwerp, and 155 lb. at Antwerp make 180 lb. at Leghorn, how many lb. at London are equal to 72 lb. at Leghorn?

Left.	*Right.*	
20	23	$20 \times 155 \times 72 = 223200$
155	180	$23 \times 180 = 4140) 223200 (53\frac{318}{414}$.
72		

Therefore $53\frac{318}{414}$ *lb. at London are equal to* 72 *lb. at Leghorn.*

2. If 12 lb. at London make 10 lb. at Amsterdam, 100 lb. at Amsterdam 120 lb. at Toulouse, how many lb. at London are equal to 40 lb. at Toulouse? *Ans.* 40.

3. If 140 braces at Venice are equal to 156 braces at Leghorn, and 7 braces at Leghorn equal to 4 ells English, how many braces at Venice are equal to 16 ells English? *Ans.* $25\frac{80}{624}$.

4. If 40 lb. at London make 36 lb. at Amsterdam, and 90 lb. at Amsterdam make 116 lb. at Dantzic, how many lb. at London are equal to 130 lb. at Dantzic? *Ans.* $112\frac{288}{4176}$.

When it is required to find how many of the last sort of coin, weight, or measure, mentioned in the question, is equal to a quantity of the first.

RULE. Place the numbers alternately, beginning at the left-hand, and let the last number stand on the right-hand; then multiply the first row for a divisor, and the second for a dividend.

Examples.

5. If 12 lb. at London make 10 lb. at Amsterdam, 100 lb. at Amsterdam 120 lb. at Toulouse, how many lb. at Toulouse are equal to 40 lb. at London? *Ans.* 40 *lb.*

6. If 40 lb. at London make 36 lb. at Amsterdam, and 90 lb. at Amsterdam 116 lb. at Dantzic, how many lb. at Dantzic are equal to 122 lb. at London? *Ans.* $141\frac{1872}{3600}$.

PROGRESSION

CONSISTS OF TWO PARTS, ARITHMETICAL AND GEOMETRICAL.

ARITHMETICAL PROGRESSION

Is the name given to a row of numbers which increase or decrease regularly by the continual adding or subtracting of equal numbers: As 1, 2, 3, 4, 5, 6, are in Arithmetical Progression by the continual increase or addition of 1; 11, 9, 7, 5, 3, 1, by the continual decrease and *subtraction of 2.*

NOTE. When any even number of terms are in Arithmetical Progression, the sum of the two extremes will be equal to the two middle numbers, or any two means equally distant from the extremes; as 2, 4, 6, 8, 10, 12 where 6 + 8, the two middle numbers are = 12 + 2, the two extremes, and = 10 + 4 the two means = 14.

When the number of terms are odd, the double of the middle term will be equal to the sum of the two extremes, or of any two means equally distant from the middle term; as 1, 2, 3, 4, 5, where the double of 3 = 5 + 1 = 2 + 4 = 6.

In Arithmetical Progression five things are to be observed, viz.:—

1. The first term, better expressed thus ..... *a.*
2. The last term ..... *l.*
3. The number of terms ..... *n.*
4. The equal difference ..... *d.*
5. The sum of all the terms ..... S.

Any three of which being given, the other two may be found.

The first, second, and third of the above five things given, to find the fifth.

RULE. Multiply the sum of the two extremes by half the number of terms, or multiply half the sum of the two extremes by the whole number of terms, the product is the total of all the terms. Or thus:—

I. *a, l, n,* are given to find S.

$$S = (a + l) \times \frac{n}{2}$$

Examples.

1. How many strokes does the hammer of a clock strike in 12 hours? Here $a = 1$, $l = 12$, $n = 12$.

12 + 1 = 13 *then* 13 × 6 = 78, *the Ans.*

2. A man buys 17 yards of cloth, and gave for the first yard 2*s.* and for the last 10*s.*, what did the 17 yards amount to? *Ans.* £5. 2*s.*

3. If 100 eggs were placed in a right line, exactly a yard asunder from one another, and the first a yard from a basket, what length of ground does that man go who gathers up these 100 eggs singly, returning with every egg to the basket to put it in?

Ans. 5 *miles* 1300 *yards.*

The first, second, and third of the five given, to find the fourth.

RULE. From the second subtract the first, the remainder divided by the third less one, gives the fourth. Or thus:—

II. *a, l, n,* are given to find *d.*

$$d = \frac{l - a}{n - 1}$$

Examples.

4. A man had eight sons, the youngest was 4 years old, and the eldest 32, they increase in Arithmetical Progression, what was the common difference of their ages? *Ans.* 4.

Here $a = 4$, $l = 32$, $n = 8$.

$32 - 4 = 28$ *then* $28 \div (8 - 1) = 4$ *common difference.*

5. A man is to travel from London to a certain place in 12 days, and to go but 3 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 58 miles, what is the daily increase, and how many miles distant is that place from London?

Ans. 5 miles daily increase.

Therefore, as 3 miles is the first day's journey,

$3 + 5 = 8$ the second day,

$8 + 5 = 13$ the third day, &c.,

The whole distance is 366 miles.

The first, second, and fourth of the five given, to find the third.

RULE. From the second subtract the first, the remainder divide by the fourth, and to the quotient add 1, and you will get the third. Or thus:—

III. a, l, d, are given to find n.

$$n = \frac{l - a}{d} + 1$$

Examples.

6. A person travelling into the country, went 3 miles the first day, and increased every day by 5 miles, till at last he went 58 miles in one day, how many days did he travel?

Here $a = 3$, $l = 58$, $d = 5$.

$58 - 3 = 55$, *then* $55 \div 5 = 11$, *and* $11 + 1 = 12$, *the number of days.*

7. A man being asked how many sons he had, said that the youngest was 4 years old, and the eldest 32, and that he increased one in his family every four years, how many had he? *Ans.* 8.

The second, third, and fourth given, to find the first.

RULE. Multiply the fourth by the third made less by 1, the product subtracted from the second gives the first. Or thus:—

IV. l, n, d *are given, to find* a.

$$a = l - d \times (n - 1).$$

Examples.

8. A man in 10 days went from London to a certain town in the country, every day's journey increasing the former by 4, and the last he went was 46 miles, what was the first? *Ans.* 10 *miles.*

Here $l = 46$, $d = 4$, $n = 10$.

$4 \times (10—1) = 36$, *then* $46—36 = 10$, *the first day's journey.*

9. A man takes out of his pocket at 8 several times, so many different numbers of shillings, every one exceeding the former by 6, the last was 46, what was the first? *Ans.* 4.

The fourth, third, and fifth given, to find the first.

RULE. Divide the fifth by the third, and from the quotient subtract half the product of the fourth multiplied by the third less 1. Or thus:—

V. n, d, S, are given, to find a.

$$a = \frac{S}{n} - \frac{d \times (n—1)}{2}$$

Examples.

10. A man is to receive £360 at 12 several payments, each to exceed the former by £4, and is willing to bestow the first payment on any one that can tell him what it is. What will that person have for his pains?

Here $S = 360$, $n = 12$, $d = 4$.

$360 \div 12 = 30$, *then* $30 - \frac{4 \times (12—1)}{2} = £8$, *the first payment.*

The first, third, and fourth given, to find the second.

RULE. Subtract the fourth from the product of the third, multiplied by the fourth, that remainder added to the first gives the second. Or thus:—

VI. a, n, d, are given, to find l.

$$l = nd—d + a, \text{ or } (n—1)\,d + a$$

Examples.

11. What is the last number of an Arithmetical Progression, beginning at 6, and continuing by the increase of 8 to 20 places?

Here $a = 6$, $d = 8$, $n = 20$.

$20 \times 8—8 = 152$, *then* $152 + 6 = 158$ the last number,

or $19 \times 8 = 152$, then $152 + 6 = 158$.

GEOMETRICAL PROGRESSION

Is a row of numbers increasing or decreasing by some common ratio; that is, by the continual multiplication or division by some constant number: as 2, 4, 8, 16, increases by the multiplier 2, and 16, 8, 4, 2, decreases by the divisor 2.

NOTE. When any number of terms is in geometrical progression, the product of the two extremes will be equal to the product of any two means, equally distant from the extremes: as 2, 4, 8, 16, 32, 64, where 64×2 are $= 4 \times 32$, $= 8 \times 16 = 128$.

When the number of terms are odd, the middle term multiplied into itself will be equal to the product of the two extremes, or that of any two means, equally distant from the middle term: as 2, 4, 8, 16, 32, where $2 \times 32 = 4 \times 16 = 8 \times 8 = 64$.

In a Geometrical Progession the same five things are to be observed as in Arithmetical, viz.:—

1. The first term.
2. The last term.
3. The number of terms.
4. The ratio, or constant multiplier or divisor.
5. The sum of all the terms.

NOTE. As a remote term in a series of numbers in Geometrical Progression is tedious to find by continual multiplications, the following contrivance for shortening the work is to be adopted:—

Write down the following row of numbers, viz., 0, 1, 2, 3, 4, 5, 6, &c. which are to be called *indices*, and underneath these write, one after the other, a few of the terms of the Geometrical series; thus—

0, 1, 2, 3, 4, 5, 6, 7, *Indices.*
1, 3, 9, 27, .. *Geom. series.*

Then to find what term would be under any advanced index,—as, for instance, what term would be under the 6, if the Geometrical series were continued,—you proceed thus: add together such indices as will make 6—thus, $1+2+3=6$; and then taking the number under any one of these for a *multiplicand*, and the numbers under the others for multipliers, multiply by them one after another, and the product will be the term required; observing, however, that if the leading term of the Geometrical series be not 1, then *each* multiplier must be *divided* by that leading term before using it.

The numbers under 1, 2, 3 above, are 3, 9, 27; therefore, taking 27 for multiplicand, you find $27 \times 9 \times 3 = 729$, which is the term that would come under 6; that is, it is the *seventh* term of the Geometrical series. *Again, $3+3=6$; therefore,* $27 \times 27 = 729$, the term required: also, *$2+2+2=6$; therefore,* $9 \times 9 \times 9 = 729$, as before.

Let it be required to find the seventh term of the Geometrical s
3, 12, 48, 192, &c.

0,	1,	2,	3,	..	..	6	*Indices.*
3,	12,	48,	192,	..	..	12288	*Geom. series.*

Indices.
3+3=6

192 *Multiplicand.*
64 *Multiplier divided by* 3, *the first term.*

768
1152

12288 *The seventh term.*

Indices.
Otherwise, 1+2+3=6

192 *Multiplicand.*
16 *Multiplier divided by* 3.

1152
192

3072
4 *Multiplier divided by* 3.

12288 *Seventh term.*

Again, let it be required to find the ninth term of the above serie

Indices.
3+3+2=8

192 *Multiplicand.*
64 *Multiplier divided by* 3.

12288
16 *Multiplier divided by* 3.

78728
12288

196608 *Ninth term.*

The indices used must always be such that their sum is 1 less thar number of the term wanted.

Examples.

1. A man agrees for 12 peaches to pay only the price of the *reckoning a farthing* for the first, a halfpenny for the second, *doubling the price* to the last, what must he give for them?

0, 1, 2, 3, 4, *Indices.*
1, 2, 4, 8, 16, *Terms.*

Indices.
For 4 + 4 + 3 = 11 *No. of terms, less* 1

16 *Term under* 4
16 4

256 8
8 3

4)2048 11

12)512

2|0)4|2 8

Ans. £2. 2*s.* 8*d.*

2. A country gentleman going to a fair to buy some oxen, meets with a person who had 23; he demanded the price of them, and was answered £16 apiece: the gentleman bids him £15 apiece, and he would buy all: the other tells him it could not be taken; but if he would give what the last ox would come to, at a farthing for the first, and doubling it to the last, he should have all. What was the price of the oxen?

Ans. £4369. 1*s.* 4*d.*

In any Geometrical Progression, not proceeding from unity, the ratio being given, to find any remote term, without producing all the intermediate terms.

3. A sum of money is to be divided among 8 persons, the first to have £20, the second £60, the third £180, and so on; what will the last have?

Ans. £43740.

0, 1, 2, 3, 540 × 540
20, 60, 180, 540, ——————— = 14580, *then* 14580 × 60 ——————— = 43740
20 20

☞ 3 + 3 + 1 = 7, *one less than the number of terms.*

4. A gentleman dying left nine sons, to whom and to his executors he bequeathed his estate in manner following:—To his executors £50, his youngest son was to have as much more as the executors, and each son to exceed the next younger by as much more; what was the eldest son's portion?

Ans. £25600.

The first term, ratio, and number of terms given, to find the sum of all the terms.

RULE. Find the last term as before, then subtract the first from it, and divide the remainder by the ratio, less one; to the quotient of which add the last term; the result will be the sum required.

Examples.

5. A servant skilled in numbers, agreed with a gentleman to serve him twelve months, provided he would give him a farthing for his first month's service, a penny for the second, 4d. for the third, and so on, *what did his wages* amount to?

Ans. £5825. 8*s.* 5¼*d.*

$256 \times 256 = 65536$, *then* $65536 \times 64 = 4194304$

0, 1, 2, 3, 4, $\qquad \dfrac{4194304-1}{4-1} = 1398101$, *then*

1, 4, 16, 64, 256,

4 + 4 + 3 = 11, *No. of terms less* 1

$1398101 + 4194304 = 5592405$ *farthings.*

6. A man bought a horse, and by agreement was to give a farthing for the first nail, three for the second, &c.; there were 4 shoes, and in each shoe 8 nails; what was the price of the horse?

Ans. £965114681693. 13*s.* 4*d.*

7. A certain person married his daughter on New-year's day, and gave her husband 1*s.* towards her portion, promising to double it on the first day of every month for one year; what was her portion?

Ans. £204. 15*s.*

8. A laceman, well versed in numbers, agreed with a gentleman to sell him 22 yards of rich gold brocade lace, for 2 pins the first yard, 6 pins the second, 9 pins the third, and so on. I desire to know what he sold the lace for, if the pins were valued at 100 for a farthing; also, what the laceman got or lost by the sale thereof, supposing the lace stood him in £7 per yard? *Ans. the lace sold for* £326886. 0*s.* 9*d.*

and he gained £326732. 0*s.* 9*d.*

PERMUTATION

Is the changing or varying the order of things in all the ways possible.

RULE. Multiply the numbers 1, 2, 3, 4, &c., together, up to the number of things to be varied: the product will be the number of changes required.

Examples.

1. How many changes may be rung upon 12 bells; and how long would they be ringing but once over, supposing 10 changes might be rung in 1 minute, and the year to contain 365 days 6 hours?

Ans. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$ *changes, which* $\div 10 = 47900160$ *minutes; and, if reduced, is* = 91 *years,* 3 *weeks,* 5 *days,* 6 *hours.*

2. A young scholar coming into town for the convenience of a good library, demands of a gentleman with whom he lodged what his diet would cost for a year; who told him £10, but the scholar not being certain what time he should stay, asked him what he must give him for so long as he should place his family (consisting of 6 persons besides himself) in different positions, every day at dinner: the gentleman thinking it would not be long, tells him £5, to which the scholar agrees. *What time* did the scholar stay with the gentleman?

Ans. 5040 days

PART II.

VULGAR FRACTIONS.

A fraction is a part or parts of a unit, as one-fourth, five-sixths, hree-eighths, &c., which are written thus: $\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, &c.

The figure above the line is called the numerator, and the under one he denominator; which shews how many parts the unit is divided into; nd the numerator shews how many of those parts are meant by the raction.

There are four sorts of Vulgar Fractions; *proper, improper, compound,* nd *mixed.*

1. A PROPER FRACTION is when the numerator is less than the denominator, as $\frac{2}{4}$, $\frac{3}{6}$, $\frac{7}{8}$, $\frac{9}{11}$, $\frac{101}{101}$, &c.

2. An IMPROPER FRACTION is when the numerator is equal to, or reater than the denominator, as $\frac{5}{5}$, $\frac{8}{4}$, $\frac{12}{12}$, $\frac{127}{3}$, &c.

3. A COMPOUND FRACTION is the fraction of a fraction, and known by he word *of*, as $\frac{1}{2}$ *of* $\frac{3}{5}$, $\frac{7}{9}$ *of* $\frac{8}{17}$ *of* $\frac{8}{12}$, &c.

4. A MIXED NUMBER OR FRACTION is composed of a whole number and fraction, as $8\frac{2}{3}$, $17\frac{1}{2}$, $8\frac{11}{97}$, &c.

REDUCTION OF VULGAR FRACTIONS.

1. *To reduce fractions to a common denominator.*

RULE 1. Multiply each numerator into all the denominators, except s own, for a new numerator; and multiply all the denominators ogether for a common denominator. Or,

2. Multiply the common denominator by the several given numerators separately, and divide the product by their several denominators, he quotients will be the new numerators.

Examples.

1. Reduce $\frac{2}{4}$ and $\frac{4}{7}$ to a common denominator. Ans. $\frac{14}{28}$, and $\frac{16}{28}$.

st numerator. *2nd numerator.*
$2 \times 7 = 14$ $4 \times 4 = 16$, *then* $4 \times 7 = 28$ den., therefore the fractions *are* $\frac{14}{28}$, *and* $\frac{16}{28}$.

2. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ to a common denominator. *Ans.* $\frac{32}{64}$, $\frac{48}{64}$, $\frac{40}{64}$.
3. Reduce $\frac{7}{8}$, $\frac{4}{9}$, $\frac{6}{10}$, and $\frac{9}{7}$ to a common denominator.
Ans. $\frac{2520}{5760}$, $\frac{2240}{5760}$, $\frac{2016}{5760}$, $\frac{2880}{5760}$.
4. Reduce $\frac{6}{10}$, $\frac{3}{4}$, $\frac{1}{7}$, and $\frac{3}{8}$ to a common denominator.
Ans. $\frac{1008}{1680}$, $\frac{840}{1680}$, $\frac{240}{1680}$, $\frac{840}{1680}$.
5. Reduce $\frac{4}{5}$, $\frac{3}{8}$, $\frac{9}{7}$, and $\frac{1}{3}$ to a common denominator.
Ans. $\frac{672}{840}$, $\frac{630}{840}$, $\frac{360}{840}$, $\frac{105}{840}$.
6. Reduce $\frac{2}{3}$, $\frac{5}{9}$, $\frac{2}{5}$, and $\frac{3}{5}$ to a common denominator.
Ans. $\frac{720}{2160}$, $\frac{1200}{2160}$, $\frac{540}{2160}$, $\frac{1296}{2160}$.

2. *To reduce a vulgar fraction to its lowest terms.*

RULE. Find a common measure by dividing the lower term by the upper, and that divisor by the remainder following, and so on, till nothing remain; the last divisor is the common measure; then divide both parts of the fraction by the common measure, and the quotient will give the fraction required.

NOTE. If the last divisor happen to be 1, the fraction is already in its lowest terms; and when a fraction hath cyphers at the right hand, it may be abbreviated by cutting them off; as $\frac{3}{4}\frac{0}{0} = \frac{3}{4}$.

Examples.

7. Reduce $\frac{24}{32}$ to its lowest terms.

24)32(1
24
———
common measure 8)24(3
24
—

then 8)$\frac{24}{32}$($= \frac{3}{4}$ *Ans.*

8. Reduce $\frac{30}{125}$ to its lowest terms. *Ans.* $\frac{6}{25}$.
9. Reduce $\frac{208}{884}$ to its lowest terms. *Ans.* $\frac{52}{171}$.
10. Reduce $\frac{192}{576}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
11. Reduce $\frac{888}{962}$ to its lowest terms. *Ans.* $\frac{37}{41}$.
12. Reduce $\frac{51184}{69312}$ to its lowest terms. *Ans.* $\frac{3}{4}$.

3. *To reduce a mixed number to an improper fraction.*

RULE. Multiply the whole number by the denominator of the fraction, and to the product add the numerator for a numerator, which place over the denominator.

NOTE. To express a whole number fraction-ways, set 1 for the denominator given.

Examples.

13. *Reduce* $18\frac{3}{7}$ *to* an improper fraction. *Ans.* $\frac{129}{7}$.

$18 \times 7 + 3 = 129$ *new numerator*, $\frac{129}{7}$.

14. Reduce $56\frac{13}{23}$ to an improper fraction. *Ans.* $^{1}\frac{2}{2}\frac{9}{3}^{1}$.
15. Reduce $183\frac{5}{21}$ to an improper fraction. *Ans.* $^{3}\frac{8}{2}\frac{4}{1}^{9}$.
16. Reduce $13\frac{4}{5}$ to an improper fraction. *Ans.* $\frac{6}{5}^{9}$.
17. Reduce $27\frac{2}{5}$ to an improper fraction. *Ans.* $^{2}\frac{4}{5}^{5}$.
18. Reduce $514\frac{5}{16}$ to an improper fraction. *Ans.* $^{8}\frac{2}{1}\frac{2}{6}^{0}$.

4. *To reduce an improper fraction to its proper terms.*

RULE. Divide the upper term by the lower.

Examples.

19. Reduce $^{1}\frac{2}{7}^{9}$ to its proper terms. *Ans.* $18\frac{3}{7}$.

$$129 \div 7 = 18\frac{3}{7}.$$

20. Reduce $^{1}\frac{2}{2}\frac{3}{3}^{5}$ to its proper terms. *Ans.* $56\frac{7}{23}$.
21. Reduce $^{3}\frac{8}{2}\frac{4}{1}^{8}$ to its proper terms. *Ans.* $183\frac{5}{21}$.
22. Reduce $\frac{6}{5}^{9}$ to its proper terms. *Ans.* $13\frac{4}{5}$.
23. Reduce $^{2}\frac{4}{5}^{5}$ to its proper terms. *Ans.* $27\frac{2}{5}$.
24. Reduce $^{8}\frac{2}{1}\frac{2}{6}^{9}$ to its proper terms. *Ans.* $514\frac{5}{16}$.

5. *To reduce a compound fraction to a single one.*

RULE. Multiply all the numerators for a new numerator, and all the denominators for a new denominator.

Reduce the new fraction to its lowest terms by Rule 2.

Examples.

25. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a single fraction.

$$2 \times 3 \times 5 = 30$$

Ans. $\frac{30}{3 \times 5 \times 8 = 120}$ — *reduced to the lowest terms* $= \frac{1}{4}$

26. Reduce $\frac{5}{6}$ of $\frac{2}{7}$ of $\frac{11}{13}$ to a single fraction. *Ans.* $\frac{220}{780} = \frac{55}{195}$.
27. Reduce $\frac{11}{12}$ of $\frac{13}{14}$ of $\frac{21}{22}$ to a single fraction. *Ans.* $\frac{3003}{4872} = \frac{143}{232}$.
28. Reduce $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{9}{10}$ to a single fraction. *Ans.* $\frac{135}{240} = \frac{9}{16}$.
29. Reduce $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{7}{9}$ to a single fraction. *Ans.* $\frac{168}{360} = \frac{7}{15}$.
30. Reduce $\frac{7}{8}$ of $\frac{5}{9}$ of $\frac{8}{10}$ to a single fraction. *Ans.* $\frac{80}{630} = \frac{8}{63}$.

6. *To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.*

RULE. Reduce the given fraction to a compound one, by comparing it with all the denominations between it, and that denomination which you would reduce it to; then reduce that compound fraction to a single one.

Examples.

31. Reduce $\frac{7}{8}$ of a penny to the fraction of a pound.

Ans. $\frac{7}{8}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{7}{1920}$.

32. *Reduce* $\frac{1}{4}$ *of a* penny to the fraction of a pound. *Ans.* $\frac{1}{960}$.

33. Reduce $\frac{4}{5}$ of a dwt. to the fraction of a lb. troy. *Ans.* $\frac{4}{1200} = \frac{1}{300}$.
34. Reduce $\frac{4}{7}$ of a lb. avoirdupoise to the fraction of an cwt.
Ans. $\frac{4}{784} = \frac{1}{196}$.

7. *To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.*

RULE. Multiply the numerator by the parts contained in the several denominations between it, and that you would reduce it to, for a new numerator, and place it over the given denominator.

Reduce the new fraction to its lowest terms.

Examples.

35. Reduce $\frac{7}{1920}$ of a pound to the fraction of a penny. *Ans.* $\frac{7}{8}$.
$7 \times 20 \times 12 = 1680$; $\frac{1680}{1920}$ *reduced to its lowest terms* $= \frac{7}{8}$.
36. Reduce $\frac{1}{960}$ of a pound to the fraction of a penny. *Ans.* $\frac{1}{4}$.
37. Reduce $\frac{4}{1200}$ of a pound troy to the fraction of a pennyweight.
Ans. $\frac{4}{5}$.
38. Reduce $\frac{4}{784}$ of an cwt. to the fraction of a lb. *Ans.* $\frac{4}{7}$.

8. *To reduce a fraction of one denomination to another of the same value, having the numerator given of the required fraction.*

RULE. As the numerator of the given fraction : is to its denominator : : so is the numerator of the intended fraction : to its denominator.

Examples.

39. Reduce $\frac{2}{3}$ to a fraction of the same value, whose numerator shall be 12. As 2 : 3 : : 12 : 18. *Ans.* $\frac{12}{18}$.
40. Reduce $\frac{5}{7}$ to a fraction of the same value, whose numerator shall be 25. *Ans.* $\frac{25}{35}$.
41. Reduce $\frac{5}{7}$ to a fraction of the same value, whose numerator shall be 47.
Ans. $\frac{47}{65\frac{4}{7}}$

9. *To reduce a fraction of one denomination to another of the same value, having the denominator given of the fraction required.*

RULE. As the denominator of the given fraction : is to its numerator :: so is the denominator of the intended fraction : to its numerator.

Examples.

42. Reduce $\frac{2}{3}$ to a fraction of the same value, whose denominator shall be 18. As 3 : 2 : : 18 : 12. *Ans.* $\frac{12}{18}$.
43. Reduce $\frac{5}{7}$ to a fraction of the same value, whose denominator *shall be 35.* Ans. $\frac{25}{35}$.

44. Reduce $\frac{5}{7}$ to a fraction of the same value, whose denominator shall be $65\frac{4}{5}$.

Ans. $\frac{47}{65\frac{4}{5}}$.

10. *To reduce a mixed fraction to a single one.*

RULE. When the integral part is in the numerator, multiply it by the denominator of the fractional part, adding in the numerator of the fractional part for a new numerator; then multiply the denominator of the fraction by the denominator of the fractional part for a new denominator.

Examples.

45. Reduce $\frac{36\frac{2}{3}}{48}$ to a simple fraction. *Ans.* $\frac{110}{144} = \frac{55}{72}$.

$36 \times 3 + 2 = 110$ *numerator.*

$48 \times 3 = 144$ *denominator.* $= \frac{55}{72}$.

46. Reduce $\frac{23\frac{5}{7}}{38}$ to a simple fraction. *Ans.* $\frac{166}{266} = \frac{83}{133}$.

When the integral part is in the denominator, multiply it by the denominator of the fractional part, adding in the numerator of the fractional part for a new denominator; then multiply the numerator of the fraction by the denominator of the fractional part for a new numerator.

Examples.

47. Reduce $\frac{47}{65\frac{4}{5}}$ to a simple fraction. *Ans.* $\frac{235}{329} = \frac{5}{7}$.

48. Reduce $\frac{19}{44\frac{1}{3}}$ to a simple fraction. *Ans.* $\frac{57}{133} = \frac{3}{7}$.

11. *To find the proper quantity of a fraction in the known parts of an integer.*

RULE. Multiply the numerator by the number of parts in one integer, and divide by the denominator.

Examples.

49. Reduce $\frac{3}{4}$ of a pound sterling to its proper quantity.

$3 \times 20 = 60$ *which* $\div 4 = 15s$. *Ans.* $15s$.

50. Reduce $\frac{2}{5}$ of a shilling to its proper quantity. *Ans.* $4\frac{4}{5}d$. $\frac{1}{5}$.

51. Reduce $\frac{5}{7}$ of a lb. avoirdupoise to its proper quantity.

Ans. 9 oz. $2\frac{2}{7}$ dr.

52. Reduce $\frac{1}{3}$ of an cwt. to its proper quantity.
Ans. 3 *qrs.*, 3 *lbs.*, 1 *oz.*, $12\frac{4}{9}$ *dr.*
53. Reduce $\frac{3}{8}$ of a lb. troy to its proper quantity. *Ans.* 7 *oz.* 4 *dwt.*
54. Reduce $\frac{3}{5}$ of an ell English to its proper quantity.
Ans. 2 *qrs.* $3\frac{1}{5}$ *nails.*
55. Reduce $\frac{4}{5}$ of a mile to its proper quantity. *Ans.* 6 *fur.* 16 *poles.*
56. Reduce $\frac{5}{8}$ of an acre to its proper quantity. *Ans.* 2 *roods*, 20 *po.*
57. Reduce $\frac{4}{7}$ of an hogshead of wine to its proper quantity.
Ans. 54 *gall.*
58. Reduce $\frac{3}{8}$ of a barrel of beer to its proper quantity.
Ans. 12 *gall.*
59. Reduce $\frac{5}{12}$ of a chaldron of coals to its proper quantity.
Ans. 15 *bush.*
60. Reduce $\frac{3}{5}$ of a month to its proper time.
Ans. 2 *weeks*, 2 *days*, $19\frac{1}{5}$ *hours.*

12. *To reduce any given quantity to the fraction of any greater denomination, retaining the same value.*

RULE. Reduce the given quantity to the lowest term mentioned for a numerator, under which set the integral part (reduced to the same term) for a denominator, and it will give the fraction required.

Examples.

61. Reduce 15*s.* to the fraction of a pound sterling. *Ans.* $£\frac{15}{20}=£\frac{3}{4}$.
62. Reduce $4\frac{3}{4}d.$ to the fraction of a shilling. *Ans.* $\frac{3}{4}$.
63. Reduce 9 oz. $2\frac{3}{4}$ dr. to the fraction of a lb. avoirdupois. *Ans.* $\frac{3}{4}$.
64. Reduce 3 qrs. 3 lb. 1 oz. $12\frac{4}{9}$ dr. to the fraction of a cwt. *Ans.* $\frac{7}{9}$.
65. Reduce 7 oz. 4 dwt. to the fraction of a lb. troy. *Ans.* $\frac{3}{8}$.
66. Reduce 2 qrs. $3\frac{1}{5}$ nails to the fraction of an English ell. *Ans.* $\frac{3}{5}$.
67. Reduce 6 fur. 16 poles to the fraction of a mile. *Ans.* $\frac{4}{5}$.
68. Reduce 2 roods 20 poles to the fraction of an acre. *Ans.* $\frac{5}{8}$.
69. Reduce 54 gallons to the fraction of a hogshead. *Ans.* $\frac{4}{7}$.
70. Reduce 12 gallons to the fraction of a barrel of beer. *Ans.* $\frac{3}{8}$.
71. Reduce 15 bushels to the fraction of a chaldron of coals. *Ans.* $\frac{5}{12}$.
72. Reduce 2 weeks, 2 days, $19\frac{1}{5}$ hours to the fraction of a month.
Ans. $\frac{3}{5}$.

ADDITION OF VULGAR FRACTIONS.

RULE. Reduce the given fractions to a common denominator, then add all the numerators together, under which place the common denominator.

Examples.

1. Add $\frac{2}{3}$ and $\frac{5}{7}$ together. *Ans.* $\frac{14}{21}+\frac{15}{21}=\frac{29}{21}=1\frac{8}{21}$.
2. Add $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{5}{8}$ together. *Ans.* $1\frac{14}{16}=1\frac{7}{8}$.

3. Add $\frac{1}{3}$, $4\frac{1}{2}$ and $\frac{3}{4}$ together. *Ans.* $4\frac{11}{12}$.
4. Add $7\frac{3}{5}$ and $\frac{2}{3}$ together. *Ans.* $8\frac{1}{15}$.
5. Add $\frac{2}{7}$ and $\frac{3}{5}$ of $\frac{7}{9}$ together. *Ans.* $1\frac{1}{7}$.
6. Add $5\frac{3}{8}$, $6\frac{7}{9}$ and $4\frac{1}{3}$ together. *Ans.* $17\frac{1}{24}$.

2. When the fractions are of several denominations, reduce them to their proper quantities of the same denomination, and add as before.

7. Add $\frac{3}{4}$ of a pound to $\frac{5}{6}$ of a shilling. *Ans.* 15*s.* 10*d.*
8. Add $\frac{1}{2}$ of a penny to $\frac{3}{5}$ of a pound. *Ans.* 13*s.* $4\frac{1}{2}$*d.*
9. Add $\frac{3}{4}$ of a pound troy to $\frac{1}{3}$ of an ounce. *Ans.* 9 *oz.* 3 *dwt.* 8 *gr.*
10. Add $\frac{2}{7}$ of a ton to $\frac{5}{8}$ of a lb. *Ans.* 16 *cwt.* 0 *qr.* 0 *lb.* 13 *oz.* $5\frac{1}{2}$ *dr.*
11. Add $\frac{3}{8}$ of a chaldron to $\frac{3}{4}$ of a bushel. *Ans.* 24 *bush.* 3 *pecks.*
12. Add $\frac{1}{3}$ of a yard to $\frac{3}{8}$ of an inch. *Ans.* 6 *inch.* 2 *bar.*

SUBTRACTION OF VULGAR FRACTIONS.

RULE. Reduce the given fractions to a common denominator, then subtract the less numerator from the greater, and place the remainder over the common denominator.

2. When the lower fraction is greater than the upper, subtract the numerator of the lower fraction from the denominator, and to that difference add the upper numerator, carrying one to the unit's place of the lower whole number.

Examples.

1. From $\frac{3}{4}$ take $\frac{5}{7}$. $3 \times 7 = 21$; $5 \times 4 = 20$; $21 - 20 = 1$ *num.* $4 \times 7 = 28$ *den.* *Ans.* $\frac{1}{28}$.
2. From $\frac{5}{6}$ take $\frac{3}{5}$ of $\frac{5}{8}$. *Ans.* $\frac{11}{24}$.
3. From $5\frac{3}{5}$ take $\frac{9}{10}$. *Ans.* $4\frac{13}{50}$.
4. From $\frac{23}{47}$ take $\frac{2}{3}$. *Ans.* $\frac{25}{141}$.
5. From $\frac{13}{20}$ take $\frac{1}{4}$ of $\frac{2}{5}$. *Ans.* $\frac{45}{160}$.
6. From $64\frac{1}{4}$ take $\frac{3}{5}$ of $\frac{3}{4}$. *Ans.* $63\frac{3}{4}$.

3. When the fractions are of several denominations, reduce them to their proper quantities, and subtract as before.

7. From $\frac{3}{4}$ of a pound take $\frac{3}{4}$ of a shilling. *Ans.* 14*s.* 3*d.*
8. From $\frac{3}{5}$ of a shilling take $\frac{1}{2}$ of a penny. *Ans.* $7\frac{1}{2}$*d.*
9. From $\frac{3}{4}$ of a lb. troy take $\frac{1}{3}$ of an ounce. *Ans.* 8 *oz.* 16 *dwts.* 16 *grs.*
10. From $\frac{4}{5}$ of a ton take $\frac{5}{8}$ of a lb. *Ans.* 15 *cwt.* 3 *qrs.* 27 *lbs.* 2 *oz.* $10\frac{3}{5}$ *dr.*
11. From $\frac{3}{5}$ of a chaldron take $\frac{3}{4}$ of a bushel. Ans. 23 bushel. 1 peck.
12. *From* $\frac{1}{4}$ *of a yard* take $\frac{3}{8}$ of an inch, Ans. 5 in. 1 barley-corn.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE. Prepare the given numbers (if they require it) by the rules of reduction: then multiply the numerators together for a new numerator, and the denominators for a new denominator.

NOTE. When any number, either whole or mixed, is multiplied by a fraction, the product will be always less than the multiplicand, in the same proportion as the multiplying fraction is less than an unit.

Examples.

1. Multiply $\frac{3}{4}$ by $\frac{4}{5}$. $3 \times 3 = 9$ *num.* $4 \times 5 = 20$ *den., therefore the product is* $\frac{9}{20}$.
2. Multiply $\frac{7}{9}$ by $\frac{2}{3}$. *Ans.* $\frac{14}{27}$.
3. Multiply $48\frac{3}{4}$ by $13\frac{5}{8}$. *Ans.* $672\frac{3}{10}$.
4. Multiply $430\frac{9}{10}$ by $18\frac{4}{7}$. *Ans.* $7935\frac{12}{35}$.
5. Multiply $\frac{16}{21}$ by $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{4}{5}$. *Ans.* $\frac{96}{294} = \frac{16}{49}$.
6. Multiply $\frac{9}{10}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. *Ans.* $\frac{3}{8}$.
7. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{1}{3}$. *Ans.* $\frac{1}{5}$.
8. Multiply $\frac{1}{4}$ of $\frac{3}{5}$ by $\frac{5}{7}$. *Ans.* $\frac{15}{294}$.
9. Multiply $5\frac{6}{7}$ by $\frac{5}{9}$. *Ans.* $4\frac{17}{63}$.
10. Multiply 24 by $\frac{2}{3}$. *Ans.* 16.
11. Multiply $\frac{3}{4}$ of 9 by $\frac{7}{8}$. *Ans.* $5\frac{23}{32}$.
12. Multiply $9\frac{1}{2}$ by $\frac{2}{3}$. *Ans.* $3\frac{1}{3}$.

DIVISION OF VULGAR FRACTIONS.

RULE. Prepare the given numbers (if they require it) by the rules of Reduction, then multiply the denominator of the divisor into the numerator of the dividend for a new numerator, and the numerator of the divisor into the denominator of the dividend for a new denominator.

NOTE. When any quantity is divided by a fraction less than unity, the quotient will be greater than the dividend: but if any quantity be divided by a number greater than unity, the quotient will be less than the dividend.

Examples.

1. Divide $\frac{9}{20}$ by $\frac{3}{5}$. $5 \times 9 = 45$ *num.* $3 \times 20 = 60$ *den., therefore the quotient is* $\frac{45}{60} = \frac{3}{4}$.
2. Divide $\frac{14}{27}$ by $\frac{2}{3}$. *Ans.* $\frac{7}{9}$.
3. Divide $672\frac{3}{10}$ by $13\frac{5}{8}$. *Ans.* $48\frac{3}{4}$.
4. Divide $7935\frac{12}{35}$ by $18\frac{4}{7}$. *Ans.* $430\frac{9}{10}$.
5. *Divide* $\frac{3}{8}$ *by* $\frac{3}{5}$ *of* $\frac{3}{4}$ *of* $\frac{5}{6}$. *Ans.* $\frac{5}{6}$.
6. *Divide* $\frac{3}{5}$ *of 16 by* $\frac{5}{7}$ *of* $\frac{3}{4}$. *Ans.* $19\frac{1}{25}$.

7. Divide $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{5}{7}$. *Ans.* $\frac{24}{25} = \frac{8}{9}$.
8. Divide $9\frac{2}{15}$ by $\frac{1}{4}$ of 7. *Ans.* $2\frac{12}{21}$.
9. Divide $\frac{2}{16}$ by $4\frac{1}{2}$. *Ans.* $\frac{1}{9}$.
10. Divide 16 by 24. *Ans.* $\frac{2}{3}$.
11. Divide $5205\frac{2}{10}$ by $\frac{1}{4}$ of 91. *Ans.* $71\frac{1}{7}$.
12. Divide $3\frac{1}{3}$ by $9\frac{1}{2}$. *Ans.* $\frac{7}{19}$.

THE SINGLE RULE OF THREE DIRECT, IN VULGAR FRACTIONS.

Rule. State the question as in the corresponding rule for integers (page 41), and then multiply the second and third terms together and divide by the first, according to the methods for multiplying and dividing fractions.

Examples.

1. If $\frac{3}{4}$ of a yard cost $\frac{5}{8}$ of a £, what will $\frac{9}{10}$ of a yard cost?

$\frac{3}{4}$: $\frac{9}{10}$ :: $\frac{5}{8}$£ : 15*s.*

$$£\frac{5}{8} \times \frac{9}{10} \div \frac{3}{4} = \frac{45}{80} \times \frac{4}{3} = \frac{180}{240} = \frac{18}{24} = \frac{3}{4}£ = \frac{3 \times 20}{4}s. = 15s.$$

2. If $\frac{5}{6}$ of a yard cost $\frac{3}{4}$ of a £, what will $\frac{1}{12}$ of a yard cost?
Ans. 14*s.* 8*d.*
3. If $\frac{3}{4}$ of a yard of lawn cost 7*s.* 3*d.*, what will $10\frac{1}{2}$ yards cost?
Ans. £4. 19*s.* $10\frac{1}{2}d.$ $\frac{8}{9}f.$
4. If $\frac{7}{8}$ of a lb. cost $\frac{3}{4}$ of a shilling, how many lb. will $\frac{8}{9}$ of a shilling buy? *Ans.* $1\frac{1}{21}$ *lb.*
5. If $\frac{3}{5}$ of an ell of Holland cost cost $\frac{1}{8}$ of a £, what will $12\frac{3}{4}$ ells cost at that rate? *Ans.* £7. 0*s.* $8\frac{3}{4}d.$ $\frac{3}{5}f.$
6. If $12\frac{1}{2}$ yards of cloth cost 15*s.* 9*d.*, what will $48\frac{1}{2}$ yards cost at the same rate? *Ans.* £3. 0*s.* $9\frac{1}{2}d.$ $\frac{4}{25}f.$
7. If $\frac{8}{10}$ of a cwt. cost 284*s.*, what will $7\frac{1}{2}$ cwt. cost at the same rate?
Ans. £118. 6*s.* 8*d.*
8. If 3 yards of broad cloth cost $£2\frac{4}{5}$, what will $10\frac{2}{7}$ yards cost?
Ans. £9. 12*s.*
9. If $\frac{1}{4}$ of a yard cost $\frac{2}{5}$ of a £, what will $\frac{3}{5}$ of an ell English come to at the same rate? *Ans.* £2.
10. If 1 lb. of cochineal cost £1. 5*s.*, what will $36\frac{7}{10}$ lb. come to?
Ans. £45. 17*s.* 6*d.*
11. If 1 yard of broad cloth cost $15\frac{5}{8}s.$, what will 4 pieces cost, each containing $27\frac{2}{7}$ yards? *Ans.* £85. 14*s.* $3\frac{1}{2}d.$ $\frac{4}{7}f.$
12. Bought $3\frac{1}{2}$ pieces of silk, each containing $24\frac{3}{5}$ ells, at 6*s.* $0\frac{3}{4}d.$ per ell, *I desire to know* what the whole quantity cost?
Ans. £25. 17*s.* $2\frac{1}{4}d.$ $\frac{3}{5}$

THE SINGLE RULE OF THREE INVERSE, IN VULGAR FRACTIONS.

Examples.

1. If 48 men can build a wall in $24\frac{1}{4}$ days, how many men can do the same in 192 days? *Ans.* $6\frac{1}{16}$ *men.**

2. If $25\frac{3}{4}s.$ will pay for the carriage of a cwt. $145\frac{1}{2}$ miles, how far may $6\frac{1}{2}$ cwt. be carried for the same money? *Ans.* $22\frac{2}{51}$ *miles.*

3. If $3\frac{1}{4}$ yards of cloth, that is $1\frac{1}{2}$ yard wide, be sufficient to make a cloak, how much must I have of that sort which is $\frac{4}{5}$ of a yard wide to make another of the same size? *Ans.* $4\frac{7}{8}$ *yards.*

4. If 3 men can do a piece of work in $4\frac{1}{2}$ hours, in how many hours will 10 men do the same work? *Ans.* $1\frac{7}{20}$ *hour.*

5. If a penny white loaf weigh 7 oz. when a bushel of wheat cost 5*s.* 6*d.*, what is the bushel worth when the penny white loaf weighs but $2\frac{1}{2}$ oz.? *Ans.* 15*s.* $4\frac{4}{7}d.$

6. What quantity of shalloon that is $\frac{3}{4}$ of a yard wide will line $7\frac{1}{2}$ yards of cloth that is $1\frac{1}{2}$ yard wide? *Ans.* 15 *yards.*

DOUBLE RULE OF THREE, IN VULGAR FRACTIONS.

Examples.

1. If a carrier receives $£2\frac{1}{10}$ for the carriage of 3 cwt. 150 miles, how much ought he to receive for the carriage of 7 cwt. $3\frac{1}{2}$ qrs. 50 miles? *Ans.* £1. 16*s.* 9*d.*

2. If £100 in 12 months gain £6 interest, what principal will $£3\frac{3}{4}$ in 9 months? *Ans.* £75.

3. If 9 students spend $£10\frac{4}{5}$ in 18 days, how much will 20 students spend in 30 days? *Ans.* £39. 18*s.* 4*d.* $\frac{240}{1485}$.

4. A man and his wife having laboured one day, earned $4\frac{2}{3}s.$, how much must they have for $10\frac{1}{2}$ days, when their two sons helped them, all being paid at the same rate. *Ans.* £4. 17*s.* $1\frac{1}{4}d.$

5. If £50 in 5 months gain $£2\frac{87}{144}$, what time will $£13\frac{1}{8}$ require to gain $£1\frac{7}{12}$? *Ans.* 9 *months.*

6. If the carriage of 60 cwt. 20 miles cost $£14\frac{1}{2}$, what weight can I have carried 30 miles for $£5\frac{7}{16}$? *Ans.* 15 *cwt.*

* This answer shows that no number of men can do it exactly—neither more nor less—in 192 days. If six men be employed, one-sixteenth of a man's work will remain to be done.

PART III.

DECIMAL FRACTIONS.

In Decimal Fractions the integer or whole thing, as one pound, one yard, one gallon, &c., is supposed to be divided into ten equal parts, and those parts into tenths, and so on without end.

So that the denominator of a decimal, being always known to consist of a unit with as many cyphers as the numerator has places, is never set down; the parts being only distinguished from the whole numbers by a point prefixed; thus ·5 which stands for $\frac{5}{10}$, ·25 for $\frac{25}{100}$, ·123 for $\frac{123}{1000}$. The prefixed point is called the decimal point.

But the different value of figures appear plainer by the following table:—

Whole numbers.							Decimal parts.					
7	6	5	4	3	2	1	·2	3	4	5	6	7
millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	parts of tens.	parts of hundreds.	parts of thousands.	parts of tens of thousands.	parts of hundreds of thousands.	parts of millions.

From which it plainly appears that, as whole numbers increase in a ten-fold proportion to the left hand, decimal parts decrease in a ten-fold proportion to the right hand: so that cyphers placed before decimal parts decrease their value, by removing them farther from the decimal point, or unit's place; thus ·5 is 5 parts of 10, or $\frac{5}{10}$; ·05 is 5 parts of 100, or $\frac{5}{100}$; ·005 is 5 parts of 1000, or $\frac{5}{1000}$; ·0005 is 5 parts of 10000, or $\frac{5}{10000}$. *But cyphers* after decimals do not alter their value. For *·5, ·50, ·500, &c.,* are each but $\frac{5}{10}$ of the unit.

A FINITE DECIMAL is that which ends at a certain number of places; but an INFINITE DECIMAL is that which nowhere ends.

A RECURRING DECIMAL is that wherein one or more figures are continually repeated, as 2·75222.

And 52·275275275 is called a COMPOUND RECURRING DECIMAL.

NOTE. In all operations, if the result ends with a series of nines, reject them, and make the next superior place a unit more; thus for 26·25999, &c., write 26·26.

In all circulating numbers, dash the first figure, as in 86·54666 = 86·546′.

ADDITION OF DECIMALS.

RULE. In setting down the proposed numbers to be added, great care must be taken to arrange them properly, so that units may be under units, tenths under tenths, &c.; and to do this there must be a due regard had to the decimal points, which ought always to stand in a direct line, one under another, and to the right hand of them carefully place the decimal parts, according to their respective values; then add them as in whole numbers.

Examples.

1. Add 72·5 + 32·071 + 2·1574 + 371·4 + 2·75. *Ans.* 480·8784.
2. Add 30·07 + 2·0071 + 59·4 + 3207·1. *Ans.* 3298·5771.
3. Add 3·5 + 47·25 + 927·01 + 2·0073 + 1·5. *Ans.* 981·2673.
4. Add 52·75 + 47·21 + 724 + 31·452 + ·3075. *Ans.* 855·7195.
5. Add 3275 + 27·514 + 1·005 + 725 + 7·32. *Ans.* 4035·839.
6. Add 27·5 + 52 + 3·2075 + ·5741 + 2720. *Ans.* 2803·2816.

SUBTRACTION OF DECIMALS.

RULE. Subtraction of decimals differs but little from whole numbers, only in placing the numbers, which must be carefully observed, as in addition of decimals.

Examples.

1. From ·2754 take ·2371. *Ans.* ·0383.
2. From 2·37 take 1·76. *Ans.* ·61.
3. From 271 take 215·7. *Ans.* 55·3.
4. *From 270·2 take 75·4075.* *Ans.* 194·7925.
5. From 571 take 54·72. *Ans.* 516·28.
6. From 625 take 76·91. *Ans.* 548·09.
7. From 23·415 take ·3742. *Ans.* 23·0408.
8. From ·107 take ·0007. *Ans.* ·1063.

MULTIPLICATION OF DECIMALS.

ULE. Place the factors, and multiply them as in whole numbers, from the product towards the right hand cut off as many places for mals as there are in both factors together; but if there should not o many places in the product, supply the defect with cyphers to the hand.

Examples.

1. Multiply ·2365 by ·2435. *Ans.* ·05758775.

Multiply 2·071 by 2·27.
Multiply 27·15 by 25·3.
Multiply 79347 by 23·15.
Multiply 17105 by ·3257.
Multiply 17105 by ·0237.
7. Multiply 27·35 by 7·70071.
8. Multiply 5·721 by ·0075.
9. Multiply ·007 by ·007.
10. Multiply 20·15 by ·2705.
11. Multiply ·907 by ·0025.

Then any number of decimals is to be multiplied by 10, 100, 1000, it is only removing the separating point in the multiplicand so y places towards the right hand as there are cyphers in the multi-r: thus, ·578 × 10 = 5·78. ·578 × 100 = 57·8. ·578 × 1000 = 578. × 10000 = 5780.

CONTRACTED MULTIPLICATION OF DECIMALS.

ULE. Put the unit's place of the multiplier under that place of the tiplicand that is intended to be kept in the product; then invert the r of all the other figures, *i.e.*, write them all the contrary way; in multiplying, begin at the figure in the multiplicand, which ds over the figure you are then multiplying with, and set down the figure of each particular product directly one under the other, and a due regard to the increase arising from the figures on the right l of that figure you begin to multiply at in the multiplicand.

OTE. That in multiplying the figure left out every time next the t hand in the multiplicand, if the product be 5, or upwards, to 15, y 1; if 15, or upwards, to 25, carry 2; and if 25, or upwards, to 35, y 3, &c.

Examples.

2. Multiply 384·672158 by 36·8345, and let there be only four es of decimals in the product. *Ans.* 14169·2065.

Contracted way.

384·672158
5438·63

115401647
23080329
3077377
115402
15387
1923

14169·2065

Common way.

384·672158
36·8345

1923|360790
15386|88632
115401|6474
3077377|264
23080329|48
115401647|4

14169206|038510

13. Multiply 3·141592 by 52·7438, and leave only four places of decimals. *Ans.* 165·6994.
14. Multiply 2·38645 by 8·2175, and leave only four places of decimals. *Ans.* 19·6107.
15. Multiply 375·13758 by 16·7324, and let there be only one place of decimals. *Ans.* 6276·9.
16. Multiply 375·13758 by 16·7324, and leave only four places of decimals. *Ans.* 6276.9520.
17. Multiply 395·3756 by ·75642, and let there be only four places of decimals. *Ans.* 299·0699.

DIVISION OF DECIMALS.

THIS rule is also worked as in whole numbers: the only difficulty is in valuing the quotient, which is done by any of the following rules.

RULE 1. The first figure in the quotient is always of the same value with that figure of the dividend, which answers to or stands over the place of units in the divisor.

2. The quotient must have always so many decimal places as the dividend has more than the divisor.

NOTE 1. If the divisor and dividend have both the same number of decimal parts, the quotient will be a whole number.

2. If the dividend has not so many places of decimals as are in the divisor, then so many cyphers must be annexed to the dividend as will make them equal, and the quotient will then be a whole number.

3. But if, when the division is done, the quotient has not so many figures as it should have places of decimals, then so many cyphers must be prefixed as there are places wanting.

Examples.

1. Divide 8564·825 by 6·321. *Ans.* 1354·9.
2. Divide 48 by 144.
3. Divide 217·75 by 65.
4. Divide 125 by ·1045.
5. Divide 709 by 2·574.
6. Divide 5·714 by 8275.
7. Divide 7382·54 by 6·4252.
8. Divide ·0851648 by 423.
9. Divide 267·15975 by 13·25.
10. Divide 72·1564 by ·1347.
11. Divide 715 by ·3075.

When numbers are to be divided by 10, 100, 1000, 10000, &c., it is performed by placing the separating point in the dividend, so many places towards the left hand as there are cyphers in the divisor.

Thus, 5784 ÷ 10 = 578·4 5784 ÷ 1000 = 5·784.
5784 ÷ 100 = 57·84 5784 ÷ 10000 = ·5784.

CONTRACTED DIVISION OF DECIMALS.

RULE. By the first rule find what is the value of the first figure in *the quotient; then* by knowing the first figure's denomination, the *decimal places may be* reduced to any number, by taking as many of

the left-hand figures of the dividend as will answer them; and in dividing omit one figure of the divisor at each following operation.

NOTE. That in multiplying every figure left out in the divisor, you must carry 1, if it be 5, or upwards, to 15; if 15, or upwards, to 25, carry 2; if 25, or upwards, to 35, carry 3, &c.

Examples.

12. Divide 721·17562 by 2·257432, and let there be only three places of decimals in the quotient.

Contracted.		Common way.	
2·257432)	721·17562(319·467	2·257432) 721·1756	2(319·467
.....	6772296	6772296	
	439460.	439460	2
	225743.	225743	2
	213717..	213717	00
	203169..	203168	88
	10548...	10548	120
	9030...	9029	728
	1518....	1518	3920
	1354....	1354	4592
	164.....	163	98280
	158.....	158	02024
	6	5	91256

13. Divide 8·758615 by 5·2714167.
14. Divide 51717591 by 8·7586.
15. Divide 25·1367 by 217·35.
16. Divide 51·47549 by ·123415.
17. Divide 70·23 by 7·9863.
18. Divide 27·104 by 3·712.

REDUCTION OF DECIMALS.

To reduce a Vulgar Fraction to a Decimal.

RULE. Add cyphers to the numerator, putting the decimal point before them, and divide by the denominator, the quotient is the decimal fraction required.

Examples.

1. Reduce ¼ to a decimal. 4)1·00(25 Ans.
2. Reduce ½ to a decimal. Ans. ·5.
3. Reduce ¾ to a decimal. Ans. ·75.

4. Reduce $\frac{3}{8}$ to a decimal. *Ans.* ·375.
5. Reduce $\frac{5}{26}$ to a decimal. *Ans.* ·1923076 +.
6. Reduce $\frac{1}{14}$ of $\frac{9}{19}$ to a decimal. *Ans.* ·6043956 +.

NOTE 1. If the given parts are of several denominations, they may be reduced either by so many distinct operations, as there are different parts, or by first reducing them into their lowest denomination, and then dividing as before ; or,

2. Bring the lowest into decimals of the next superior denomination, and on the right-hand of the decimal found place the parts given of the next superior denomination; so proceeding till you bring out the decimal parts of the highest integer required, by still dividing the product by the next superior denominator. (*See Ex.* 11 *below.*)

7. Reduce 5*s.* to the decimal of a £. $\frac{5}{20} = ·25$. *Ans.* ·25£.
8. Reduce 9*s.* to the decimal of a £. *Ans.* ·45£.
9. Reduce 16*s.* to the decimal of a £. *Ans.* ·8£.
10. Reduce 8*s.* 4*d.* to the decimal of a £. *Ans.* ·4166.
11. Reduce 16*s.* $7\frac{3}{4}$*d.* to the decimal of a £. *Ans.* ·8322916£.

By Note 1.	By Note 2.
16*s.* $7\frac{3}{4}$*d.*	4)3·00
12	12)7·75
199	2|0)16·64583′
4	·8322916′
960)799(·8322916′	

12. Reduce 19*s.* $5\frac{1}{2}$*d.* to the decimal of a £. *Ans.* ·972916.
13. Reduce 12 grains to the decimal of a lb. troy. *Ans.* ·002083.
14. Reduce 12 drams to the decimal of a lb. avoirdupoise. *Ans.* ·046875.
15. Reduce 2 qrs. 14 lb. to the decimal of an cwt. *Ans.* ·625.
16. Reduce 2 furlongs to the decimal of a league. *Ans.* ·0833.
17. Reduce 2 quarts 1 pint to the decimal of a gallon. *Ans.* ·625.
18. Reduce 4 gallons, 2 quarts of wine, to the decimal of an hogshead. *Ans.* ·071428 +.
19. Reduce 2 gallons 1 quart of beer to the decimal of a barrel. *Ans.* ·0625.
20. Reduce 52 days to the decimal of a year. *Ans.* ·142465 +.

To find the value of any Decimal Fraction, in the known parts of an Integer.

RULE. Multiply the decimal given by the number of parts of the next inferior denomination, cutting off the decimals from the product ; then multiply these decimals by the number of parts of the next inferior denomination; thus proceeding till you have brought in the parts of the lowest denomination.

Examples.

21. What is the value of ·8322916 of a £. ? *Ans.* 16*s.* 7¾*d.*

20

16·6458320 shillings.
12

7·7499840 pence.
4

2·9999360 farthings = 3 farthings.

22. What is the value of ·002084 of a lb. troy ? *Ans.* 12·00384 *qrs.*
23. What is the value of ·046875 of a lb. avoirdupoise ? *Ans.* 12 *drs.*
24. What is the value of ·625 of an cwt. *Ans.* 2 *qrs.* 14 *lb.*
25. What is the value of ·625 of a gallon ? *Ans.* 2 *quarts* 1 *pint.*
26. What is the value of ·071428 of a hogshead of wine ?
Ans. 4 *gallons* 1 *quart* ·999856.
27. What is the value of ·0625 of a barrel of beer ? *Ans.* 2 *gall.* 1 *qt.*
28. What is the value of ·142465 of a year ? *Ans.* 51·999725 *days.*

DECIMAL TABLES OF COIN, WEIGHT, AND MEASURE.

TABLE I.

ENGLISH COIN, £1 the Integer.

Sh.	Dec.	Sh.	Dec.
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

Pence.	Decimals.
6	·025
5	·020833
4	·016666
3	·0125
2	·00833
1	·00416

Farthings.	Decimals.
3	·003125
2	·0020833
1	·0010416

TABLE II.

ENGLISH COIN, 1*s.* LONG MEASURE, 1 ft. the Integer.

Pence or Inches.	Decimals.
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Farthings.	Decimals.
3	·0625
2	·041666
1	·020833

TABLE III.

TROY WEIGHT, 1 lb. the Integer.

Ounces the same as Pence in the last Table.

Pennywts.	Decimals.
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Grains.	Decimals.
12	·002083
11	·001910
10	·001736
9	·001562

Grains.	Decimals.
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 oz. the Integer.

Pennyweights the same as Shillings in the first Table.

Grains.	Decimals.
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125
5	·010416
4	·008333
3	·00625
2	·004166
1	·002083

TABLE IV.

AVOIRDUPOISE WEIGHT,

112 lb. the Integer.

Qrs.	Decimals.
3	·75
2	·5
1	·25

Pounds.	Decimals.
14	·125
13	·116071
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428

Pounds.	Decimals.
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Ounces.	Decimals.
8	·004464
7	·003906
6	·003348
5	·002790
4	·002232
3	·001674
2	·001116
1	·000558

Qr. oz.	Decimals.
3	·000418
2	·000279
1	·000139

TABLE V.

AVOIRDUPOISE WEIGHT,

1 lb. the Integer.

Ounces.	Decimals.
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

Drams.	Decimals.
8	·03125
7	·027343
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.

LIQUID MEASURE,

1 tun the Integer.

Gallons.	Decimals.
100	·396825
90	·357141
80	·317460
70	·277777
60	·238095
50	·198412
40	·158730
30	·119047
20	·079365
10	·039682
9	·035714
8	·031746
7	·027777
6	·023809
5	·019841
4	·015873
3	·011904
2	·007936
1	·003968

Pints.	Decimals.
4	·001984
3	·001488
2	·000992
1	·000496

A hogshead the Integer.

Gallons.	Decimals.
30	·476190
20	·317460
10	·158730
9	·142857
8	·126984
7	·111111
6	·095238
5	·079365
4	·063492
3	·047619
2	·031746
1	·015873

Pints.	Decimals.
3	·005952
2	·003968
1	·001984

TABLE VII.

LIQUID MEASURE,
1 gallon the Integer.

DRY MEASURE,
1 quarter the Integer.

Pints.	Decim.	Bush.
4	·5	4
3	·375	3
2	·25	2
1	·125	1

Qr. pts.	Decim.	Pecks.
3	·09375	3
2	·0625	2
1	·03125	1

Decimals.	Qr. pecks.
·0234375	3
·015625	2
·0078125	1

Decimals.	Pints.
·005859	3
·003906	2
·001953	1

TABLE VIII.

LONG MEASURE.

1 mile the Integer.

Yards.	Decimals.
1000	·568182
900	·511364
800	·454545
700	·397727
600	·340909
500	·284091
400	·227272
300	·170454
200	·113636
100	*·056818*
90	·051136
80	·045454
70	·039773
60	·034091
50	·028409
40	·022727
30	·017045
20	·011364
10	·005682
9	·005114
8	·004545
7	·003977
6	·003409
5	·002841
4	·002273
3	·001704
2	·001136
1	·000568

Feet.	Decimals.
2	·0003787
1	·0001894

Inches.	Decimals.
6	·0000947
3	·0000474
1	·0000158

TABLE IX.

TIME.

1 year the Integer.

Months the same as Pence in the second Table.

Days.	Decimals.
365	1·000000
300	·821918
200	·547945
100	·273973
90	·246575
80	·219178
70	·191781
60	·164383
50	·136986
40	·109589
30	·082192
20	·054794
10	·027397
9	·024657
8	·021918
7	·019178
6	·016438
5	·013698
4	·010959
3	·008219
2	·005479
1	·002739

1 day the Integer.

Hours.	Decimals.
12	·5
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041666

Minutes.	Decimals.
30	·020833
20	·013888
10	·006944
9	·00625
8	·005555
7	·004861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001388
1	·000694

TABLE X.

CLOTH MEASURE, 1 yd. the Integer.

Qrs. same as Table 4.

Nails.	Decimals.
2	·125
1	·0625

TABLE XL.

LEAD WEIGHT, a fother the Integer.

Hund.	Decimals.
10	·512820
9	·461538
8	·410256
7	·358974
6	·307692
5	·256410
4	·205128
3	·153846
2	·102564
1	·051282

Qrs.	Decimals.
2	·025641
1	·012820

Pounds.	Decimals.
14	·0064102
13	·0059523
12	·0054945
11	·0050366
10	·0045787
9	·0041208
8	·0036630
7	·0032051
6	·0027472
5	·0022893
4	·0018315
3	·0013736
2	·0009157
1	·0004578

THE RULE OF THREE IN DECIMALS.

Examples.

1. If 26½ yards cost £3. 16*s.* 3*d.*, what will 32¼ yards come to?

Ans. £4. 12*s.* 9½*d.*

Yards.		Yards.		£.	
26·5	:	32·25	::	3·8125	:
				32·25	

26·5)122·953125(4·63974 = £4. 12*s.* 9½*d.*

2. What will the pay of 540 men come to at £1. 5*s.* 6*d.* per man?

Ans. £688. 10*s.*

3. If 7¾ yards of cloth cost £2. 12*s.* 9*d.*, what will 140¼ yards of the same cost? *Ans.* £47. 16*s.* 3*d.* 2·4 *far.*

4. If a chest of sugar, weighing 7 cwt. 2 qrs. 14 lb., cost £36. 12*s.* 9*d.*, what will 2 cwt. 1 qr. 21 lb. of the same cost?

Ans. £11. 14*s.* 2*d.* 3·5 *far.*

5. A grocer buys 24 ton 12 cwt. 2 qrs. 14 lb. 12 oz. of tobacco for £3678. 6*s.* 4*d.*, what will 1 oz. come to? *Ans.* 1*d.*

6. What will 326¼ lb. of tobacco come to, when 1½ lb. is sold for 3*s.* 6*d.*? *Ans.* £38. 1*s.* 3*d.*

7. What is the worth of 19 oz. 3 dwt. 5 gr. of gold, at £2. 19*s.* per oz.? *Ans.* £56. 10*s.* 5*d.* 2·99 *far.*

8. What is the worth of 827¾ yards of painting, at 10½*d.* per yard?

Ans. £36. 4*s.* 3*d.* 1·5 *far.*

9. If I lent my friend £34 for ⅞ of a year, how much ought he to lend me $\frac{5}{12}$ of a year to requite my kindness? *Ans.* £51.

10. If ¾ of a yard of cloth, that is 2¼ yards broad, make a garment, how much that is ⅔ of a yard wide will make the same?

Ans. 2·109375 *yards.*

11. If 1 ounce of silver cost 5*s.* 6*d.*, what is the price of a tankard that weighs 1 lb. 10 oz. 10 dwt. 4 grs.? *Ans.* £6. 3*s.* 9*d.* 2·2 *far.*

12. *If 1 lb. of tobacco* cost 15*d.*, what cost 3 hogsheads, weighing *together 16 cwt. 1* qr. 19 lb.? Ans. £107. 13*s.* 9

13. If 1 cwt. of currants cost £2. 9*s.* 6*d.*, what will 45 cwt. 3 qrs. 14 lb. cost at the same rate? *Ans.* £113. 10*s.* 9¾*d.*

14. Bought 6 chests of sugar, each 6 cwt. 3 qrs., at £2. 16*s.* per cwt., what do they come to? *Ans.* £113. 8*s.*

15. Bought a tankard for £10. 12*s.*, at the rate of 5*s.* 4*d.* per oz., what was the weight? *Ans.* 39 *oz.* 15 *dwt.*

16. Gave £187. 3*s.* 3*d.* for 25 cwt. 3 qrs. 14 lb. of tobacco, at what rate did I buy it at per lb.? *Ans.* 15½*d.*

17. Bought 29 lb. 4 oz. of coffee for £10. 11*s.* 3*d.*, what is the value of 3 lb.? *Ans.* £1. 1*s.* 8*d.*

18. If I gave 1*s.* 1*d.* for 3¼ lb. of cheese, what will be the value of 1 cwt.? *Ans.* £1. 14*s.* 8*d.*

EXTRACTION OF THE SQUARE ROOT.

EXTRACTING the Square Root is to find out such a number as being multiplied into itself, the product will be equal to the given number.

RULE 1. Point the given number, that is, put a dot over the unit's place, then over the hundreds, and so over every second figure throughout. The number will thus be separated into periods.

2. Seek the greatest square number in the first period towards the left-hand, placing the square number under the first period, and the root thereof in the quotient; subtract the square number from the first period, and to the remainder bring down the next period, and call that the resolvend.

3. Double the quotient, and place it for a divisor on the left-hand of the resolvend; seek how often the divisor is contained in the resolvend (reserving always the unit's place), and put the answer in the quotient, and also on the right-hand side of the divisor; then multiply by the figure last put in the quotient, and subtract the product from the resolvend; bring down the next period to the remainder (if there be any more) and proceed as before.

ROOTS,	1	2	3	4	5	6	7	8	9
SQUARES,	1	4	9	16	25	36	49	64	81

Examples.

1. What is the square root of 119025? *Ans.* 345.

```
119025(345
  9
-----
64)290
   256
-----
685)3425
    3425
```

2. What is the square root of 106929? *Ans.* 327.
3. What is the square root of 2268741? *Ans.* 1506·23 +.
4. What is the square root of 7596796? *Ans.* 2756·228 +.
5. What is the square root of 36372961? *Ans.* 6031.
6. What is the square root or 22071204? *Ans.* 4698.

When the given number consists of a whole number, and decimals together, make the number of decimals even by adding cyphers to them, so that a point may be over the unit's place of the whole number.

7. What is the square root of 3271·4207? *Ans.* 57·19 +.
8. What is the square root of 4795·25731? *Ans.* 69·247 +.
9. What is the square root of 4·372594? *Ans.* 2·091 +.
10. What is the square root of 2·2710957? *Ans.* 1·50701 +.
11. What is the square root of ·00032754? *Ans.* ·01809 +.
12. What is the square root of 1·270054? *Ans.* 1·1269 +.

To extract the Square Root of a Vulgar Fraction.

RULE. Reduce the fraction to its lowest terms: then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator.

If the fraction be a surd, *i.e.*, a number where a root can never be exactly found, reduce it to a decimal, and then extract the root.

Examples.

13. What is the square root of $\frac{3204}{5184}$? *Ans.* $\frac{8}{9}$.
14. What is the square root of $\frac{2704}{4225}$? *Ans.* $\frac{4}{5}$.
15. What is the square root of $\frac{9216}{12544}$? *Ans.* $\frac{9}{7}$.

SURDS.

16. What is the square root of $\frac{375}{640}$? *Ans.* ·89934 +.
17. What is the square root of $\frac{357}{475}$? *Ans.* ·86602 +.
18. What is the square root of $\frac{473}{540}$? *Ans.* ·93309 +.

To Extract the Square Root of a Mixed Number.

RULE. Reduce the fractional part of the mixed number to its lowest term, and then the mixed number to an improper fraction.

2. Extract the roots of the numerator and denominator for a new numerator and denominator.

If the mixed number given be a surd, reduce the fractional part to a *decimal*, annex it to the whole number, and extract the square root *therefrom.*

Examples.

19. What is the square root of $51\frac{1}{25}$? *Ans.* $7\frac{1}{5}$.
20. What is the square root of $27\frac{9}{16}$? *Ans.* $5\frac{1}{4}$.
21. What is the square root of $4\frac{4}{49}$? *Ans.* $3\frac{1}{7}$.

SURDS.

22. What is the square root of $85\frac{1}{4}$? *Ans.* 9·27 +.
23. What is the square root of $8\frac{5}{9}$? *Ans.* 2·9519 +.
24. What is the square root of $6\frac{3}{5}$? *Ans.* 2·5819 +.

The Application.

1. An army consists of 331776 men: if these are closely placed together in rows so as to form a square, how many men will there be in a side of that square? [Men so placed are in rank and file.] *Ans.* 576.

2. In a certain pavement which is exactly square, there are 9409 square feet: what is the length of each side of it? *Ans.* 97 *feet.*

To find a mean proportional between any two given numbers.

RULE. The square root of the product of the given numbers is the mean proportional sought.

Examples.

27. What is the mean proportional between 3 and 12?
$3 \times 12 = 36$ *then* $\sqrt{36} = 6$ *the mean proportional.*
28. What is the mean proportional between 4276 and 842?
Ans. 1897·4 +.

To find the side of a square equal in area to any given superficies.

RULE. The square root of the content of any given superficies is the square equal sought.

Examples.

29. If the content of a given circle be 160, what is the side of the square equal to it? *Ans.* 12·6491+.
30. If the area of a circle is 750, what is the side of the equal square?
Ans. 27·38612+.

The area of a circle given, to find the diameter.

RULE. As 355 : 452, or, as 1 : 1·273239 :: so is the area to the square of the diameter. Or, multiply the square root of the area by 1·12837, and the product will be the diameter.

Examples.

31. What length of cord will be fit to tie to a cow's tail, the other end fixed in the ground, to let her have liberty of eating an acre of grass and no more, supposing the cow and tail to be $5\frac{1}{2}$ yards.
Ans. 6·136 *perches.*

The area of a circle given, to find the periphery, or circumference.

RULE. As 113 : 1420, or, as 1 : 12·56637 :: the area to the square *of the periphery. Or,* multiply the square root of the area by 3·5449, *and the product is the* circumference.

Examples.

32. When the area is 12, what is the circumference? *Ans.* 12·279+.
33. When the area is 160, what is the periphery? *Ans.* 44·839+.

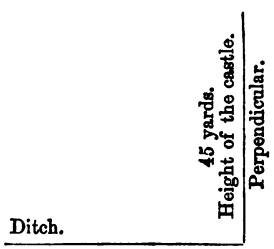
Any two sides of a right angled triangle given, to find the third side.

1. *The base and perpendicular given, to find the hypothenuse.*

RULE. The square root of the sum of the squares of the base and perpendicular is the length of the hypothenuse, or longest side.

Examples.

34. The top of a castle, which from the ground is 45 yards high, is surrounded with a ditch 60 yards broad; what length must a ladder be to reach from the outside of the ditch to the top of the castle?
Ans. 75 *yards.*



35. The wall of a town is 25 feet high, and is surrounded by a moat of 30 feet in breadth: I desire to know the length of a ladder that will reach from the outside of the moat to the top of the wall.
Ans. 39·05 *feet.*

The hypothenuse and perpendicular given, to find the base.

RULE. The square root of the difference of the squares of the hypothenuse and perpendicular is the length of the base.

The base and hypothenuse given, to find the perpendicular.

RULE. The square root of the difference of the squares of the hypothenuse and base is the height of the perpendicular.

N.B. The two last questions may be varied for examples to the two last propositions.

Any number of men being given, to form them into a square, or to find the number of rank and file.

RULE. The square root of the number of men given, is the number of men either in rank or file. (*See Ex.* 1, *p.* 113).

36. An army consisting of 331776 men, I desire to know how many *rank and file?* Ans. 576.

37. A certain square pavement contains 48841 square stones, all of the same size, I demand how many are contained in one of the sides?

Ans. 221.

EXTRACTION OF THE CUBE ROOT.

To extract the Cube Root, is to find out a number, which being multiplied into itself, and then into that product, produces the given number.

RULE 1. Point every third figure of the cube given, beginning at the unit's place; seek the greatest cube in the first period, and subtract it therefrom; put the root in the quotient, and bring down the figures in the next period to the remainder for a RESOLVEND.

2. Find a DIVISOR by multiplying the square of the quotient by 3. See how often it is contained in the resolvend, rejecting the units and tens, and put the answer in the quotient.

3. To find the SUBTRAHEND.—1. Cube the last figure in the quotient. 2. Multiply all the figures in the quotient by 3, except the last, and that product by the square of the last. 3. Multiply the divisor by the last figure. Add these products together, to get the subtrahend, which subtract from the resolvend; to the remainder bring down the next period, and proceed as before.

ROOTS,	1	2	3	4	5	6	7	8	9
SQUARES,	1	8	27	64	125	216	343	512	729

Examples.

1. What is the cube root of 99252847?

99252847(463
64 = *cube of* 4

Divisor.
Square of 4 × 3 = 48)35252 *resolvend.*

216 = *cube of* 6.
432 = 4 × 3 × *square of* 6.
288 = *divisor* × 6

33336 *subtrahend.*

Divisor.
Square of 46 × 3 = 6348)1916847 *resolvend.*

27 = *cube of* 3.
1242 = 46 × 3 × *square of* 3.
19044 = *divisor* × 3.

1916847 *subtrahend.*

ANOTHER AND MORE CONCISE METHOD OF EXTRACTING THE CUBE ROOT.

RULE 1. Point every third figure of the cube given, beginning at the unit's place, then find the highest cube in the first period, and subtract it therefrom, put the root in the quotient, bring down the figures in the next point to the remainder for a resolvend.

2. Square the quotient and triple the square for a divisor, as $4 \times 4 \times 3 = 48$. Find how often it is contained in the resolvend, rejecting units and tens, and put the answer in the quotient.

3. Square the last figure in the quotient, and put it on the right-hand of the divisor.

As $6 \times 6 = 36$ put the divisor = 4836.

4. Triple the last figure in the quotient and multiply by the former, put the result under the other, units under tens, add them together, and multiply the sum by the last figure in the quotient, subtract that product from the resolvend, bring down the next period, and proceed as before.

Examples.

1. What is the cube root of 99252847?

Square of 4 × 3 = 48 *divisor.*
Square of 6 *put to* 48 = 4836
6 × 3 × 4 = 72

5556 × 6 =
Square of 46 = 2116 × 3 = 6348 *divisor.*
Square of 3 = 9 *put to* 6348 = * 634809
3 × 3 × 46 = 414

638949 × 3 = 1916847.

99252847(463
64

35252
33336

1916847

2. What is the cube root of 389017? *Ans.* 73.
3. What is the cube root of 5735339? *Ans.* 179.
4. What is the cube root of 32461759? *Ans.* 319.
5. What is the cube root of 84604519? *Ans.* 439.
6. What is the cube root of 259694072? *Ans.* 638.
7. What is the cube root of 48228544? *Ans.* 364.
8. What is the cube root of 27054036008? *Ans.* 3002.
9. What is the cube root of 22069810125? *Ans.* 2805.
10. What is the cube root of 122615327232? *Ans.* 4968.
11. What is the cube root of 219365327791? *Ans.* 6031.
12. What is the cube root of 673373097125? *Ans.* 8765.

* *When the quotient is* 2 *or* 3, there must be a cypher put to supply the place of tens.

When the given number consists of a whole number and decimals together, make the number of decimals consist of 3, 6, 9, &c., places by adding cyphers thereto, so that there may be a point over the unit's place of the whole number.

13. What is the cube root of 12·977875? *Ans.* 2·35.
14. What is the cube root of 36155·027576? *Ans.* 33·06+.
15. What is the cube root of ·001906624? *Ans.* ·124.
16. What is the cube root of 33·230979637? *Ans.* 3·215+.
17. What is the cube root of 15926·972504? *Ans.* 25·16+.
18. What is the cube root of ·053157376? *Ans.* ·376.

To extract the cube root of a vulgar fraction.

RULE. Reduce the fraction to its lowest terms, then extract the cube root of the numerator and denominator for a new numerator and denominator; but if the fraction be a surd, reduce it to a decimal, and then extract the root from it.

Examples.

19. What is the cube root of $\frac{343}{648}$? *Ans.* $\frac{7}{9}$.
20. What is the cube root of $\frac{324}{1500}$? *Ans.* $\frac{3}{5}$.
21. What is the cube root of $\frac{1458}{3136}$? *Ans.* $\frac{9}{14}$.

SURDS.

22. What is the cube root of $\frac{7}{9}$? *Ans.* ·829+.
23. What is the cube root of $\frac{5}{9}$? *Ans.* ·822+.
24. What is the cube root of $\frac{2}{3}$? *Ans.* ·873+.

To extract the cube of a mixed number.

RULE. Reduce the fractional part to its lowest terms, and then the mixed number to an improper fraction, extract the cube roots of the numerator and denominator for a new numerator and denominator; but if the mixed number given be a surd, reduce the fractional part to decimals, annex it to the whole number, and extract the root therefrom.

Examples.

25. What is the cube root of $12\frac{19}{27}$? *Ans.* $2\frac{1}{3}$.
26. What is the cube root of $31\frac{15}{343}$? *Ans.* $3\frac{1}{7}$.
27. What is the cube root of $405\frac{88}{125}$? *Ans.* $7\frac{3}{5}$.

SURDS.

28. What is the cube root of $7\frac{1}{5}$? Ans. 1·93+.
29. What is the cube root of $9\frac{1}{8}$? Ans. 2·092+.
30. What is the cube root of $8\frac{5}{7}$? Ans. 2·051+.

The Application.

1. If a cubical piece of timber be 47 inches long, 47 inches broad, and 47 inches deep, how many cubical inches doth it contain?
Ans. 103823.

2. There is a cellar dug that is 12 feet every way, in length, breadth, and depth, how many solid feet of earth was taken out of it?
Ans. 1728.

3. There is a stone of a cubic form, which contains 389017 solid feet, what is the superficial content of one of its sides? *Ans.* 5329.

Between two numbers given, to find two mean proportionals.

RULE. Divide the greater extreme by the less, and the cube root of the quotient multiplied by the less extreme gives the less mean; multiply the said cube root by the less mean, and the product will be the greater mean proportional.

Examples.

4. What are the two mean proportionals between 6 and 162?
Ans. 18 *and* 54.

5. What are the two mean proportionals between 4 and 108?
Ans. 12 *and* 36.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

RULE. The cube root of the solid content of any solid body given, is the side of the cube of equal solidity.

Examples.

6. If the solid content of a globe is 10648, what is the side of a cube of equal solidity. *Ans.* 22.

The side of the cube being given, to find the side of the cube that shall be double, treble, &c. in quantity to the cube given.

RULE. Cube the side given, and multiply it by 2, 3, &c.; the cube root of the product is the side sought; or multiply the side given by the square root of 2, of 3, &c.

Example.

7. There is a cubical vessel whose side is 12 inches, and it is required to find the side of another vessel that is to contain three times as much.
Ans. 17·306.

EXTRACTING OF THE BIQUADRATE ROOT.

To extract the Biquadrate Root is to find out a number, which, being *involved four times into* itself, will produce the given number.

RULE. First extract the square root of the given number, and then extract the square root of that square root, and it will give the biquadrate root required.

Examples.

1. What is the biquadrate (or fourth power) of 27? *Ans.* 531441.
2. What is the biquadrate of 76? *Ans.* 33362176.
3. What is the biquadrate of 275? *Ans.* 5719140625.
4. What is the biquadrate root of 531441? *Ans.* 27.
5. What is the biquadrate root of 33362176? *Ans.* 76.
6. What is the biquadrate root of 5719140625? *Ans.* 275.

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

1. Prepare the number given for extraction, by pointing off from the unit's place as the root required directs.

2. Find the first figure in the root, and subtract the proposed power of it from the given number.

3. Bring down the first figure in the next period to the remainder, and call it the Dividend.

4. Involve the root into the next inferior power to that which is given, multiply it by the given power, and call it the Divisor.

5. Find a quotient figure by common division, and annex it to the root; then involve the whole root to the given power, and call that the Subtrahend.

6. Subtract that number from as many periods of the given power as are brought down, beginning at the lowest place, and to the remainder bring down the first figure of the next period for a new dividend.

7. Find a new divisor, and proceed in all respects as before.

Examples.

1. What is the square root of 141376?

```
141376(376
 9                3 ×   2 = 6 divisor.
 -               37 ×  37 = 1369 subtrahend.
6)51 dividend.   37 ×   2 = 74 divisor.
 --             376 × 376 = 141376 subtrahend.
 1369 subtrahend.
 ----
74)  447 dividend.
 ----
 141376 subtrahend.
 ======
```

2. What is the cube root of 53157376?

53157376(376
27

27)261 *dividend.*

50653 *subtrahend.*

4107)25043 *dividend.*

53157376 *subtrahend.*

3 × 3 × 3 = 27 *divisor.*
37 × 37 × 37 = 50653 *subtrahend.*
37 × 37 × 3 = 4107 *divisor.*
376 × 376 × 376 = 53157376 *subtrahend.*

3. What is the biquadrate root of 19987173376?

19987173376(376
81

108)1188 *dividend.*

1874161 *subtrahend.*

202612)1245563 *dividend.*

19987173376 *subtrahend.*

3 × 3 × 3 × 4 = 108 *divisor.*
37 × 37 × 37 × 37 = 1874161 *subtrahend.*
37 × 37 × 37 × 4 = 202612 *divisor.*
376 × 376 × 376 × 376 = 19987173376 *subtrahend.*

SIMPLE INTEREST.

THERE are five things to be observed in Simple Interest, viz. :—

p. the Principal.
t. the Time, in years.
r. the Ratio, or interest of £1 for 1 year.
i. the whole Interest.
a. the Amount.

A TABLE OF RATIOS.

Per cent.		Per cent.		Per cent.	
3	·03	5½	·055	8	·08
3½	·035	6	·06	8½	·085
4	·04	6½	·065	9	·09
4½	·045	7	·07	9½	·095
5	·05	7½	·705	10	·1

NOTE. The ratio is the simple interest of £1 for one year, at the rate per cent. proposed, and is found thus :—

As £100 : £3 :: £1 : ·03. As 100 : 3·5 :: 1 : ·035.

When the principal, time, and rate per cent. are given, to find the interest.

RULE. Multiply the principal, time, and ratio together, and it will give the interest required.

NOTE. The proposition and rule are better expressed thus :—

I. *When p, t, r, are given, to find i.*

RULE. $prt = i$.

NOTE. When two or more letters are put together like a word, they are to be multiplied one into another.

Examples.

1. What is the interest of £945. 10*s.* for three years, at 5 per cent. per annum? *Ans.* £945·5 × ·05 × 3 = 141·825 *or* £141. 16*s.* 6*d.*
2. What is the interest of £547. 14*s.*, at 4 per cent. per annum, for 6 years? *Ans.* £131. 8*s.* 11½*d.* ·08*f.*
3. What is the interest of £796. 15*s.* at 4½ per cent. per annum, for 5 years. *Ans.* £179. 5*s.* 4½*d.*
4. What is the interest of £397. 9*s.* 5*d.* for 2½ years, at 3½ per cent. per annum? *Ans.* £34. 15*s.* 6*d.* 3·5472*f.*
5. What is the interest of £554. 17*s.* 6*d.* for 3 years 8 months, at 4½ per cent. per annum? *Ans.* £91. 11*s.* 1½*d.*
6. What is the interest of £236. 18*s.* 8*d.* for 3 years 8 months, at 5½ per cent. per annum? *Ans.* £47. 15*s.* 7*d.* 2·293*f.*

When the interest is for any number of days only.

RULE. Multiply the interest of £1 for 1 day, at the given rate, by the principal and number of days, it will give the answer.

INTEREST OF £1 FOR ONE DAY.

Per cent.		Per cent.	
3	·00008219178	6½	·00017808219
3½	·00009589041	7	·00019178082
4	·00010958904	7½	·00020547945
4½	·00012328767	8	·00021917808
5	·00013698630	8½	·00023287671
5½	·00015068493	9	·00024657534
6	·00016438356	9½	·00026027397

NOTE. The above table is thus found :—

As 365 : ·03 :: 1 : ·00008219178. And as 365 : ·035 :: 1 : ·00009589041, &c.

Examples.

7. What is the interest of £240 for 120 days, at 4 per cent. pe annum? *Ans.* ·0010958904 × 240 × 120 = £3. 3*s.* 1¼*d*

8. What is the interest of £563, at 6 per cent. per annum, for 12 days? *Ans.* £11. 13*s.* 2½*d*

9. What is the interest of £560 for 60 days, at 5 per cent. pe annum? *Ans.* £4. 12*s.* 0½*d*

10. What is the interest of £364. 18*s.* for 154 days, at 5 per cent. pe annum? *Ans.* £7. 13*s.* 11¼*d*

11. What is the interest of £725. 15*s.* for 74 days, at 4 per cent. pe annum? *Ans.* £5. 17*s.* 8½*d*

12. What is the interest of £100 from the 1st of June, 1855, to th 9th of March following, at 5 per cent. per annum?

Ans. £3. 16*s.* 11¾*d*

II. *When p, r, t, are given, to find a.*

RULE. $prt + p = a$.

Examples.

13. What will £279. 12*s.* amount to in 7 years, at 4½ per cent. pe annum? *Ans.* £367. 13*s.* 5*d.* 3·04.

279·6 × ·045 × 7 + 279·6 = 367·674.

14. What will £320. 17*s.* amount to in 5 years, at 3½ per cent. pe annum? *Ans.* £376. 19*s.* 11*d.* 2·8.

15. What will £679. 13*s.* amount to in 6 years, at 5 per cent. pe annum? *Ans.* £883. 10*s.* 10*d.* 3·2.

When there is any odd time given with the whole years, reduce th odd time into days, and work with the decimal parts of a year whic are equal to those days.

16. What will £926. 12*s.* amount to in 5¼ years, at 4 per cent. pe annum? *Ans.* £1130. 9*s.* 1·92.

17. What will £368. 16*s.* amount to in 7¾ years, at 6½ per cent. pe annum? *Ans.* £554. 11*s.* 7*d.* 3·68.

18. What will £273. 18*s.* amount to in 4 years 175 days, at 3 pe cent. per annum? *Ans.* £310. 14*s.* 1*d.* 3·35080064.

III. *When a, r, t, are given, to find p.*

RULE. $\frac{a}{rt + 1} = p$.

Examples.

19. What principal, being put to interest, will amount to £367. 13 5*d.* 3·04 *f.* in 7 years, at 4½ per cent. per annum?

·045 × 7 + 1 = 1·315, *then* 367·674 ÷ 1·315 = £279. 12

20. What principal, being put to interest, will amount to £376. 19*s.* 11*d.* 2·8*f.* in 5 years, at 3½ per cent. per annum? *Ans.* £320. 17*s.*

21. What principal, being put to interest, will amount to £883. 10*s.* 10*d.* 3·2*f.* in 6 years, at 5 per cent. per annum? *Ans.* £679. 13*s.*

22. What principal, being put to interest, will amount to £1130. 9*s.* 0*d.* 1·92*f.* in 5½ years, at 4 per cent. per annum? *Ans.* £926. 12*s.*

23. What principal will amount to £554. 11*s.* 7*d.* 3·68*f.* in 7¾ years, at 6½ per cent. per annum? *Ans.* £368. 16*s.*

24. What principal will amount to £310. 14*s.* 1*d.* 3·35080064*f.* in 4 years 175 days, at 3 per cent. per annum? *Ans.* £273. 18*s.*

IV. *When a, p, t, are given, to find r.*

RULE. $\frac{a-p}{pt.} = r.$

Examples.

25. At what rate per cent. will £279. 12*s.* amount to £367. 13*s.* 5*d.* 3·04*f.* in 7 years? 367·674 − 279·6 = 88·074. 279·6 × 7 = 1957·2, *then* 88·074 ÷ 1957·2 = ·045 *or* 4½ *per cent.*

26. At what rate per cent. will £320. 17*s.* amount to £376. 19*s.* 11*d.* 2·8*f.* in 5 years? *Ans.* 3½ *per cent.*

27. At what rate per cent. will £679. 13*s.* amount to £883. 10*s.* 10*d.* 3·2*f.* in 6 years? *Ans.* 5 *per cent.*

28. At what rate per cent. will £926. 12*s.* amount to £1130. 9*s.* 0*d.* 1·92*f.* in 5½ years? *Ans.* 4 *per cent.*

29. At what rate per cent. will £368. 16*s.* amount to £554. 11*s.* 7*d.* 3·68*f.* in 7¾ years? *Ans.* 6½ *per cent.*

30. At what rate per cent. will £273. 18*s.* amount to £310. 14*s.* 1*d.* 3·35080064*f.* in 4 years 175 days? *Ans.* 3 *per cent.*

V. *When a, p, r, are given, to find t.*

RULE. $\frac{a-p}{pr.} = t.$

Examples.

31. In what time will £279. 12*s.* amount to £367. 13*s.* 5*d.* 3·04*f.*, at 4½ per cent.? 367·674 − 279·6 = 88·074. 279·6 × ·045 = 12·5820, *then* 88·074 ÷ 5820 = 7 *years.*

32. In what time will £320. 17*s.* amount to £376. 19*s.* 11*d.* 2·8*f.*, at 3½ per cent.? *Ans.* 5 *years.*

33. In what time will £679. 13*s.* amount to £883. 10*s.* 10*d.* 3·2*f.*, at 5 per cent.? *Ans.* 6 *years.*

34. In what time will £926. 12*s.* amount to £1130. 9*s.* 0*d.* 1·92*f.*, at 4 per cent.? *Ans.* 5½ *years.*

35. In what time will £368. 16*s.* amount to £554. 11*s.* 7*d.* 3·68*f.*, at 6½ per cent.? *Ans.* 7¾ *years.*

36. In what time will £273. 18*s.* amount to £310. 14*s.* 1*d.* 3·35080064*f.* at 3 per cent.? *Ans.* 4 *years* 175 *days.*

ANNUITIES, PENSIONS, ETC., IN ARREARS.

Annuities or pensions, &c., are said to be in arrears when they ar payable or due, either yearly, half-yearly, or quarterly, and are unpai for any number of payments.

NOTE. u represents the annuity, pension, or yearly rent. t, r, a, s before.

I. *u, r, t, are given, to find a.*

RULE. $\frac{tru-tu}{2} \times r + tu = a.$ Or, $\frac{1}{2}\{(t-1)r+2\} \times tu = a.$

Examples.

37. If a salary of £150 be forborne 5 years, at 5 per cent., wh would it amount to? *Ans.* £82

$(5 \times 5 \times 150) - 5 \times 150 = 3000$, *then* $\frac{3000}{2} \times \cdot 05 + 5 \times 150 = £825.$

Or, $4 \times \cdot 05 + 2 = 2 \cdot 2$, *then* $1 \cdot 1 \times 5 \times 150 = 825.$

38. If £250 yearly pension be forborne 7 years, what will it amou to in that time at 6 per cent.? *Ans.* £206

39. There is a house let upon a lease for 5½ years, at £60 per annum what will be the amount for the whole time at 4½ per cent.?

Ans. £363. 8*s.* 3*d*

40. Suppose an annual pension of £28 remain unpaid for 8 year what would it amount to at 5 per cent.? *Ans.* £263. 4

NOTE. When the annuities, &c., are to be paid half-yearly or qua terly, then

For half-yearly payments, take half of the ratio, half of the annuity &c., and twice the number of years; and,

For quarterly payments, take a fourth part of the ratio, a fourth par of the annuity, &c., and four times the number of years, and work a before.

Examples.

41. If a salary of £150, payable every half-year, remains unpaid fo 5 years, what would it amount to in that time at 5 per cent.?

Ans. £834. 7*s.* 6*d*

42. If a salary of £150, payable every quarter, was left unpaid for years, what would it amount to in that time at 5 per cent.?

Ans. £839. 1*s.* 3*d*

NOTE. It may be observed, by comparing these last examples, that th amount of the half-yearly payments are more advantageous than th yearly, and the quarterly more than the half-yearly.

II. *When a, r, t, are given, to find u.*

RULE. $\frac{2a}{tir - tr + 2t} = u.$ Or, $\frac{2a}{\{2 + (t-1)r\} \times t} = u.$

Examples.

13. If a salary amounted to £825 in 5 years, at 5 per cent., what was salary? *Ans.* £150.

$5 \times 2 = 1650$, $5 \times 5 \times \cdot05 - 5 \times \cdot05 + 5 \times 2 = 11$, *then* $1650 \div 11 = £150$.

Or, $(2 + 4 \times \cdot05) \times 5 = 11$; *then* $1650 \div 11 = 150$.

14. If a house is to be let upon a lease for $5\frac{1}{2}$ years, and the amount that time be £363. 8*s.* 3*d.*, at $4\frac{1}{2}$ per cent., what is the yearly rent? *Ans.* £60.

15. If a pension amounted to £2065 in 7 years, at 6 per cent., what s the pension? *Ans.* £250.

16. Suppose the amount of a pension be £263. 4*s.* in 8 years, at 5 · cent., what is the pension? *Ans.* £28.

NOTE. When the payments are half-yearly, then take $4a$ and half of ı ratio, and twice the number of years: and if quarterly, then take one-fourth of the ratio, and four times the number of years, and ıceed as before.

17. If the amount of a salary, payable half-yearly, for 5 years and at er cent., be £834. 7*s.* 6*d.*, what is the salary? *Ans.* £150.

18. If the amount of an annuity, payable quarterly, be £839. 1*s.* 3*d.* 5 years, at 5 per cent., what is the annuity? *Ans.* £150.

III. *When u, a, t, are given, to find r.*

RULE. $\dfrac{2a - 2ut}{utt - ut} = r.$ Or, $2\dfrac{a - ut}{ut(t-1)} = r.$

Examples.

19. If a salary of £150 per annum amount to £825 in 5 years, what he rate per cent.? *Ans. 5 per cent.*

$825 \times 2 - 150 \times 5 \times 2 = 150$, *then* $\dfrac{150}{150 \times 5 \times 5 - 150 \times 5} = \cdot05$.

$825 - 150 \times 5 = 75$, *and* $750 \times 4 = 3000$; *then twice* $\frac{75}{3000} = \frac{15}{300} = \cdot05$.

20. If a house be let upon lease for $5\frac{1}{2}$ years, at £60 per annum, and amount for that time be £363. 8*s.* 3*d.*, what is the rate per cent.? *Ans.* $4\frac{1}{2}$ *per cent.*

21. If a pension of £250 per annum amounts to £2065 in 7 years, at is the rate per cent.? *Ans.* 6 *per cent.*

22. Suppose the amount of a yearly pension of £28 be £263. 4*s.* in 8 rs, what is the rate per cent.? *Ans.* 5 *per cent.*

NOTE. When the payments are half-yearly, take $4a - 4ut$ for a divi- d, and work with half the annuity, and double the number of years a divisor; if quarterly, take $8a - 8ut$, and work with a fourth of annuity, and four times the number of years.

23. If a salary of £150 per annum, payable half-yearly, amounts to 34. 7*s.* 6*d.* in 5 years, what is the rate per cent.? Ans. 5 per cent.

54. If an annuity of £150 per annum, payable quarterly, amounts to £839. 1*s.* 3*d.* in five years, what is the rate per cent.? *Ans.* 5 *per cent.*

IV. *When u, a, r, are given, to find t.*

RULE. Put $\frac{1}{r} - \cdot 5 = x$; *then* $\sqrt{(\frac{2a}{ur} + xx)} - x = t$.

Examples.

55. In what time will a salary of £150 per annum amount to £825, at 5 per cent.? *Ans.* 5 *years.*

$$x = \frac{1}{\cdot 05} - \cdot 5 = 19 \cdot 5 \text{; then } \sqrt{(\frac{825 \times 2}{150 \times \cdot 05} + 19 \cdot 5 \times 19 \cdot 5)} - 19 \cdot 5 = 5 \text{ years.}$$

56. If a house is let upon lease for a certain time for £60 per annum, and the amount be £363. 8*s.* 3*d.*, at 4½ per cent., what time was it let for? *Ans.* 5½ *years.*

57. If a pension of £250 per annum, being forborne a certain time, amounts to £2065 at 6 per cent., what was the time of forbearance? *Ans.* 7 *years.*

58. In what time will a yearly pension of £28 amount to £263. 4*s.*, at 5 per cent.? *Ans.* 8 *years.*

NOTE. If the payments are half-yearly, take half the ratio and half the annuity; if quarterly, one-fourth of the ratio and one-fourth of the annuity; and t will be equal to those half-yearly or quarterly payments.

59. If an annuity of £150 per annum, payable half-yearly, amounts to £834. 7*s.* 6*d.*, at 5 per cent., what time was the payment forborne? *Ans.* 5 *years.*

60. If a yearly pension of £150, payable quarterly, amounts to £839. 1*s.* 3*d.*, at 5 per cent., what was the time of forbearance? *Ans.* 5 *years.*

PRESENT WORTH OF ANNUITIES.

NOTE. p represents the present worth; u, t, r, as before.

I. *When u, t, r, are given, to find p.*

RULE. $\frac{ttr - tr + 2t}{2tr + 2} \times u = p$.

Examples.

61. What is the present worth of £250 per annum, to continue 5 years, at 5 per cent.? *Ans.* 660.

$5 \times 5 \times \cdot 05 - 5 \times \cdot 05 + 5 \times 2 = 11$. $5 \times \cdot 05 \times 2 + 2 = 2 \cdot 5$,
then $(11 + 2 \cdot 5) \times 150 =$ £660.

62. What is the rent of a house of £60 yearly, to continue $5\frac{1}{2}$ years, worth in ready money, at $4\frac{1}{2}$ per cent.? *Ans.* £291. 6*s.* 3*d.*

63. What is the present worth of £250 per annum, to continue 7 years, at 6 per cent.? *Ans.* £1454. 4*s.* 6*d.*

64. What is a pension of £28 per annum worth in ready money, at 5 per cent. for 8 years? *Ans.* £188.

NOTE. The same thing is to be observed as in the first rule of annuities in arrears, concerning half-yearly and quarterly payments.

65. What is the present worth of £150 payable half-yearly, for 5 years, at 5 per cent.? *Ans.* £667. 10*s.*

66. What is the present worth of £150 payable quarterly, for 5 years, at 5 per cent.? *Ans.* £671. 5*s.*

NOTE. By comparing the last examples, it will be found that the present worth of half-yearly payments is more advantageous than yearly; and quarterly than half-yearly.

II. *When p, t, r, are given, to find u.*

RULE. $\dfrac{tr+1}{ttr-tr+2t} \times 2p = u$

Examples.

67. If the present worth of a salary be £660, to continue 5 years, at 5 per cent., what was the salary? *Ans.* £150.

$5 \times \cdot05 + 1 = 1\cdot25$, *and* $5 \times 5 \times \cdot05 - 5 \times \cdot05 + 10 = 11$.

then $\dfrac{1\cdot25}{11} \times 660 \times 2 = 150$

68. There is a house let upon lease for $5\frac{1}{2}$ years to come; I desire to know the yearly rent, when the present worth, at $4\frac{1}{2}$ per cent., is £291. 6*s.* 3*d.*? *Ans.* £60.

69. What annuity is that which for 7 years' continuance, at 6 per cent., produces £1454. 4*s.* 6*d.* present worth? *Ans.* £250.

70. What annuity is that which for 8 years' continuance produces £188 for the present worth, at 5 per cent.? *Ans.* £28.

NOTE. When the payments are half-yearly, take half the ratio, twice the number of years, and multiply by $4p$; and when quarterly, take one-fourth of the ratio, four times the number of years, and multiply by $8p$.

71. There is an annuity, payable half-yearly, for 5 years to come; what is the yearly rent, when the present worth, at 5 per cent., is £667. 10*s.*? *Ans.* £150.

72. There is an annuity, payable quarterly, for 5 years to come; I desire to know the yearly income, when the present worth, at 5 per cent., is £671. 5*s.*? *Ans.* £150.

III. *When u, p, t, are given, to find r.*

RULE. $\frac{(ut - p) \times 2}{2pt + ut - utt} = r.$ Or, $\frac{(ut - p)\,2}{2pt - (t-1)\,ut} = r.$

Examples.

73. At what rate per cent. will an annuity of £150 per annum, to continue 5 years, produce the present worth of £660? *Ans. 5 per cent.*

$(150 \times 5 - 660) \times 2 = 180$; $2 \times 660 \times 5 + 150 \times 5 - 150 \times 5 \times 5 = 3600$; *then* $180 \div 3600 = \cdot 05 = 5$ *per cent.*

Or, $(150 \times 5 - 660)\,2 = 180$; $2 \times 660 \times 5 - 4 \times 150 \times 5 = 3000$, *then* $180 \div 3600 = \cdot 05$.

74. If a yearly rent of £60 per annum, to continue $5\frac{1}{2}$ years, produce £291. 6*s.* 3*d.* for the present worth, what is the rate per cent.? *Ans.* $4\frac{1}{2}$ *per cent.*

75. If an annuity of £250 per annum, to continue 7 years, produce £1454. 4*s.* 6*d.* for the present worth, what is the rate per cent.? *Ans.* 6 *per cent.*

76. If a pension of £28 per annum, to continue 8 years, produces £188 for the present worth, what is the rate per cent.? *Ans. 5 per cent.*

NOTE. When the annuities, or rents, &c. are to be paid half-yearly, or quarterly, then

For half-yearly payments, take half of the annuity, &c. and twice the number of years, the quotient will be the ratio of half the rate per cent.; and,

For quarterly payments, take a fourth part of the annuity, &c. and four times the number of years, the quotient will be the ratio of a fourth part of the rate per cent.

77. If an annuity of £150 per annum, payable half-yearly, having 5 years to come, is sold for £667. 10*s.*, what is the rate per cent.? *Ans. 5 per cent.*

78. If an annuity of £150 per annum, payable quarterly, having 5 years to come, is sold for £671. 5*s.*, what is the rate per cent.? *Ans. 5 per cent.*

IV. *When u, p, r, are given, to find t.*

RULE. $\frac{2}{r} - \frac{2p}{u} - 1 = x$; *then* $\sqrt{\left(\frac{2p}{ur} + \frac{xx}{4}\right)} - \frac{x}{2} = t.$

Examples.

79. If an annuity of £150 per annum produce £660 for the present worth, at 5 per cent., what is the time of its continuance? *Ans. 5 years.*

$$\frac{2}{\cdot 05} - \frac{660 \times 2}{150} - 1 = 30\cdot 2\,; \quad \frac{660 \times 2}{150 \times \cdot 05} = 176\,;$$

$$\frac{30\cdot 2 \times 20\cdot 2}{4} = 228\cdot 01\,; \textit{ then } \sqrt{228\cdot 01 + 176} = 20\cdot 1$$

$$20\cdot 1 - \frac{3\cdot 02}{2} = 5 \textit{ years.}$$

80. For what time may a salary of £60 be purchased for £291. 6*s.* 3*d.* at 4½ per cent.? *Ans.* 5½ *years.*

81. For how long time may £250 per annum be purchased for £1454. 4*s.* 6*d.* at 6 per cent.? *Ans.* 7 *years.*

82. What time may a pension of £28 per annum be bought for £188 at 5 per cent.? *Ans.* 8 *years.*

NOTE. When the payments are half-yearly, then u will be equal to the half annuity, &c.; r, half the ratio; and t, the number of payments; and

When the payments are quarterly, u will be equal to a fourth part of the annuity, &c.; r, the fourth of the ratio; and t, the number of payments.

83. If an annuity of £150 per annum, payable half-yearly, is sold for £667. 10*s.* at 5 per cent., I desire to know the number of payments, and the time to come? *Ans.* 10 *payments, 5 years.*

84. An annuity of £150 per annum, payable quarterly, is sold for £671. 5*s.* at 5 per cent., what is the number of payments, and time to come? *Ans.* 20 *payments, 5 years.*

ANNUITIES, &c. TAKEN IN REVERSION.

1. *To find the present worth of an annuity, &c. taken in reversion.*

RULE. 1. Find the present worth of the yearly sum at the given rate, and for the time of its continuance, thus,

$$\frac{ttr - tr + 2t}{2tr + 2} : \times u = p.$$

2. Change p into a, and find what principal being put to interest will amount to a at the same rate, and for the time to come before the annuity, &c. commences, thus,

$$\frac{a}{tr + 1} = p.$$

Examples.

85. What is the present worth of an annuity of £150 per annum, to continue 5 years, but not to commence till the end of 4 years, allowing 5 per cent. to the purchaser? *Ans.* £550.

$$\frac{5 \times 5 \times \cdot 05 - 5 \times \cdot 05 + 2 \times 5}{5 \times \cdot 05 \times 2 + 2} \times 150 = 660, \textit{ then}$$

$$\frac{5 \times 5 \times \cdot 05 - 5 \times \cdot 05 + 2 \times 5}{5 \times \cdot 05 \times 2 + 2} = 4\cdot 4 \times 150 = \frac{660}{4 \times \cdot 05 + 1} = 550$$

86. What is the present worth of a lease of £50 per annum, to continue 4 years, but is not to commence till the end of 5 years, allowing 4 per cent. to the purchaser? *Ans.* £152. 5*s.* 11¾*d.*

87. A person having the promise of a pension of £20 per annum, for 8 years, but not to commence till the end of four years, is willing to dispose of the same at 5 per cent., what will be the present worth?

Ans. £111. 18*s.* 1·14*d.*

88. A legacy of £40 being left for 6 years to a person of 15 years of age, but is not to commence till he is 21, he wanting money, is desirous of selling the same at 4 per cent., what is the present worth?

Ans. £171. 14*s.*

2. *To find the yearly income of an annuity, &c. in reversion.*

RULE. 1. Find the amount of the present worth at the given rate, and for the time before the reversion, thus, $ptr + p = a$

2. Change a into p, and find what annuity being sold will produce p at the same rate, and for the time of its continuance, thus, $\frac{tr + 1}{ttr - tr + 2t} \times 2p = u.$

Examples.

89. A person having an annuity left him for 5 years, which does not commence till the end of 4 years, disposed of it for £550, allowing 5 per cent. to the purchaser, what was the yearly income? *Ans.* £150.

$$550 \times 4 \times \cdot05 + 550 = 660; \frac{5 \times \cdot05 + 1,}{5 \times 5 \times \cdot05 - 5 \times \cdot05 + 5 \times 2} =$$

$$\cdot113636 \times 660 \times 2 = £150.$$

90. There is a lease of a house taken for 4 years, but not to commence till the end of 5 years, the lessee would sell the same for £152. 6*s.* present payment, allowing 4 per cent. to the purchaser, what is the yearly rent? *Ans.* £50.

91. A person having the promise of a pension for 8 years, which does not commence till the end of 4 years, has disposed of the same for £111. 18*s.* 1·14*d.* present money, allowing 5 per cent. to the purchaser, what was the pension? *Ans.* £20.

92. There is a certain legacy left to a person of 15 years of age, which is to be continued for 6 years, but not to commence till he arrives to the age of 21; he wanting a sum of money, sells it for £171. 14*s.*, allowing 4 per cent. to the buyer, what was the annuity left him?

Ans. £40.

REBATE, OR DISCOUNT.

NOTE. s represents the sum to be discounted.
p the present worth.
t the time.
r the ratio.

I. *When s, t, r, are given, to find p.*

RULE. $\dfrac{s}{tr+1} = p.$

Examples.

1. What is the present worth of £357. 10*s.*, to be paid 9 months hence, at 5 per cent.? *Ans.* £344. 11*s.* 6*d.* 3·168*f.*

Here £357. 10s. = £375·5, and 9 *mo.* = ·75*y.*

$$\frac{357{\cdot}5}{{\cdot}75 \times {\cdot}05 + 1} = 344{\cdot}5783.$$

2. What is the present worth of £275. 10*s.*, due 7 months hence, at 5 per cent.? *Ans.* £267. 13*s.* 10·164*d.*

3. What is the present worth of £875. 5*s.* 6*d.*, due 5 months hence, at 4½ per cent.? *Ans.* £859. 3*s.* 3*d.* 3·2544*f.*

4. How much ready money can I receive for a note of £75, due 15 months hence, at 5 per cent.? *Ans.* £70. 11*s.* 9·1764*d.*

II. *When p, t, r, are given, to find s.*

RULE. $ptr + p = s.$ Or, $(tr + 1) \times p = s.$

Examples.

5. If the present worth of a sum of money, due 9 months hence, allowing 5 per cent., be £344. 11*s.* 6*d.* 3·168*f.*, what was the sum first due? *Ans.* £357. 10*s.*

$$344{\cdot}5783 \times {\cdot}75 \times {\cdot}05 + 344{\cdot}5783 = £357.\ 10s.$$

Or, $(75 \times {\cdot}05 + 1) \times 344{\cdot}5783 = 357{\cdot}5.$

6. A person owing a certain sum, payable 7 months hence, agrees with the creditor to pay him down £267. 13*s.* 10·164*d.*, allowing 5 per cent. for present payment, what is the debt? *Ans.* £275. 10*s.*

7. A person receives £859. 3*s.* 3*d.* 3·254*f.* for a sum of money due 5 months hence, allowing the debtor 4½ per cent. for present payment, what was the sum due? *Ans.* £875. 5*s.* 6*d.*

8. A person paid £70. 11*s.* 9·1764*d.*, for a debt due 15 months hence, he being allowed 5 per cent. for the discount, how much was the debt? *Ans.* £75.

III. *s, p, t, are given, to find r.*

RULE. $\frac{s-p}{tp} = r.$ Or, $(\frac{s}{p} - 1) \div t.$

Examples.

9. At what rate per cent. will £357. 10*s.*, payable 9 months hence, produce £344. 11*s.* 6*d.* 3·168*f.* for present payment? *Ans.* 5 *per cent.*

$$\frac{357{\cdot}5-344{\cdot}5783}{344{\cdot}5783 \times {\cdot}75} = {\cdot}05 = 5 \textit{ per cent.}$$

10. At what rate per cent. will £275. 10*s.*, payable 7 months hence, produce £267. 13*s.* 10·164*d.* for present payment? *Ans.* 5 *per cent.*

11. At what rate per cent. will £875. 5*s.* 6*d.*, payable 5 months hence, produce the present payment of £859. 3*s.* 3*d.* 3·2544*f.*?

Ans. $4\frac{1}{2}$ *per cent.*

12. At what rate per cent. will £75, payable 15 months hence, produce the present payment of £70. 11*s.* 9·1764*d.*? *Ans.* 5 *per cent.*

IV. *When s, p, r, are given, to find t.*

RULE. $\frac{s-p}{rp} = t.$ Or, $(\frac{s}{p} - 1) \div r.$

Examples.

13. The present worth of £357. 10*s.*, due for a certain time to come, is £344. 11*s.* 6*d.* 3·168*f.*, at 5 per cent., in what time should the sum have been paid without any rebate? *Ans.* 9 *months.*

$$\frac{357{\cdot}5-344{\cdot}5783}{344{\cdot}5783 \times {\cdot}05} = {\cdot}75 = 9 \textit{ months.}$$

14. The present worth of £275. 10*s.*, due for a certain time to come, is £267. 13*s.* 10·164*d.*, at 5 per cent., in what time should the sum have been paid without any rebate? *Ans.* 7 *months.*

15. A person receives £859. 3*s.* 3*d.* 3·2544*f.* for £875. 5*s.* 6*d.*, due at a certain time to come, allowing $4\frac{1}{2}$ per cent. discount; I desire to know in what time the debt should have been discharged without any rebate? *Ans.* 5 *months.*

16. I have received £70. 11*s.* 9·1752*d.* for a debt of £75, allowing the person 5 per cent. for prompt payment, I desire to know when the debt would have been payable without the rebate. *Ans.* 15 *months.*

EQUATION OF PAYMENTS.

To find the equated time for the payment of a sum of money due at several times.

RULE. Find the present worth of each payments for its respective time, thus ..... $\frac{s}{1+tr} = p.$

Add all the present worths together, and put P for the sum. Also put S for the sum of all the payments, then $\frac{S-P}{Pr}$ = the equated time.

Examples.

1. D owes E £200, whereof £40 is to be paid at 3 months, £60 at 6 months, and £100 at 9 months; at what time may the whole debt be paid together, rebate being made at 5 per cent.?

Ans. 6 *months* 26 *days.*

Here the payments being made at $\frac{1}{4}$ year, $\frac{1}{2}$ year, and $\frac{3}{4}$ year, the several values of tr are $\cdot25 \times \cdot05$, $\cdot5 \times \cdot05$, and $\cdot75 \times \cdot05$. And by the rule

$$\frac{40}{1 + \cdot25 \times \cdot05} = \frac{40}{1\cdot0125} = 39\cdot5061$$

$$\frac{60}{1 + \cdot5 \times \cdot05} = \frac{60}{1\cdot025} = 58\cdot5365$$

$$\frac{100}{1 + \cdot75 \times \cdot05} = \frac{100}{1\cdot0375} = 96\cdot3855$$

$$194\cdot4281 = P$$
$$200 = S$$

$Pr = 9\cdot721405)\ 5\cdot5719 = S-P$ (·57315 years)
= 6 months 26 days, the equated time.

2. D owes F £800, whereof £200 is to be paid in 3 months, £200 at 4 months, and £400 at 6 months; but they agreeing to make but one payment of the whole, at the rate of 5 per cent. rebate, the true equated time is demanded? *Ans.* 4 *months* 22 *days.*

3. E owes F. £1200, which is to be paid as follows:—£200 down, £500 at the end of 10 months, and the rest at the end of 20 months; but they agreeing to have one payment of the whole, rebate at 3 per cent., the true equated time is demanded? *Ans.* 1 *year* 11 *days.*

COMPOUND INTEREST.

THE letters made use of in Compound Interest are:—

a the Amount.
p the Principal.
t the Time.
r the Amount of £1 for 1 year, at any given rate, which is thus found:—

As 100 : 105 :: 1 : 1·05. As 100 : 105·5 :: 1 : 1·055.

A TABLE OF THE AMOUNT OF ONE POUND FOR ONE YEAR.

Rates per cent.	Amounts of £1.	Rates per cent.	Amounts of £1.	Rates per cent.	Amounts of £1.
3	1·03	5½	1·055	8	1·08
3½	1·035	6	1·06	8½	1·085
4	1·04	6½	1·065	9	1·09
4½	1·045	7	1·07	9½	1·095
5	1·05	7½	1·075	10	1·1

A TABLE showing the amount of £1 for any number of years under 31, at 5 and 6 per cent. per annum. [NOTE. This table gives the value of r^t in several of the following examples.]

Years.	Rates 5	Rates 6	Years.	Rates 5	Rates 6
1	1·05000	1·06000	16	2·18287	2·54035
2	1·10250	1·12360	17	2·29201	2·69277
3	1·15762	1·19101	18	2·40662	2·85434
4	1·21550	1·26247	19	2·52695	3·02559
5	1·27628	1·33822	20	2·65329	3·20713
6	1·34009	1·41852	21	2·78596	3·39956
7	1·40710	1·50363	22	2·92526	3·60353
8	1·47745	1·59384	23	3·07152	3·81975
9	1·55132	1·68948	24	3·22510	4·04893
10	1·62889	1·79084	25	3·38635	4·29187
11	1·71034	1·88929	26	3·55567	4·54938
12	1·79585	2·01219	27	3·73345	4·82234
13	1·88565	2·13292	28	3·92013	5·11168
14	1·97993	2·26090	29	4·11613	5·41838
15	2·07892	2·39655	30	4·32194	5·74349

NOTE. The above table is thus made:—As 100 : 105 : : 1 : 1·05 for the first year; then, As 100 : 1·05 : : 105 : 1·025, second year, &c.

I. *When p, t, r, are given, to find a.*

RULE. $p \times r^t = a$. r^t may be taken from the above table, when the rate is 5 or 6 per cent.

Examples.

1. What will £225 amount to in 3 years time, at 5 per cent. per annum? *Ans.* 1·05 × 1·05 × 1·05 = 1·157625 *then* 1·157625 × 225 = £260. 9*s.* 3¾*d.*

2. What will £200 amount to in 4 years at 5 per cent. per annum? *Ans.* £243. 2·025*s.*

3. What will £450 amount to in 5 years, at 4 per cent. per annum? *Ans.* £547. 9*s.* 10*d.* 2·0538368*f.*

4. What will £500 amount to in 4 years, at 5½ per cent. per annum? *Ans.* £619. 8*s.* 2*d.* 3·8323*f.*

II. *When a, r, t, are given, to find p.*

RULE. $\frac{a}{r^t} = p.$

Examples.

5. What principal being put to interest will amount to £260. 9*s.* 3¾*d.* in 3 years, at 5 per cent. per annum?

$$1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 = 1{\cdot}157625. \quad \frac{260{\cdot}465625}{1{\cdot}157625} = £225.$$

6. What principal being put to interest will amount to £243. 2·025*s.* in four years, at 5 per cent. per annum? *Ans.* £200.

7. What principal will amount to £547. 9*s.* 10*d.* 2·0538368*f.* in 5 years, at 4 per cent. per annum? *Ans.* £450.

8. What principal will amount to £619. 8*s.* 2*d.* 3·8323*f.* in 4 years, at 5½ per cent.? *Ans.* £500.

III. *When p, a, t, are given, to find r.*

RULE. $\frac{a}{p} = r^t$ which being extracted by the rule of extraction (the time given in the question showing the power), will give *r*.

Examples.

9. At what rate per cent. will £225 amount to £260. 9*s.* 3¾*d.* in 3 years? *Ans.* 5 *per cent.*

$\frac{260{\cdot}465625}{225} = 1{\cdot}157625$, the cube root of which (it being the 3rd power) = 1·05 = 5 *per cent.*

10. At what rate per cent. will £200 amount to £243. 2·025*s.* in 4 years? *Ans.* 5 *per cent.*

11. At what rate per cent. will £450 amount to £547. 9*s.* 10*d.* 2·0538368*f.* in 5 years? *Ans.* 4 *per cent.*

12. At what rate per cent. will £500 amount to £619. 8*s.* 2*d.* 3·8323*f.* in 4 years? *Ans.* 5½ per cent.

IV. *When p, a, r, are given, to find t.*

RULE. $\frac{a}{p} = r^t$ which being continually divided by r as often as possible, the number of those divisions will be equal to t.

Examples.

13. In what time will £225 amount to £260. 9*s.* 3¾*d.*, at 5 per cent.?

$$\frac{260{\cdot}465625}{225} = 1{\cdot}157625; \quad \frac{1{\cdot}157625}{1{\cdot}05} = 1{\cdot}1025; \quad \frac{1{\cdot}1025}{1{\cdot}05} = 1{\cdot}05; \quad \frac{1{\cdot}05}{1{\cdot}05} = 1$$

As there have been three divisions by 1·05, the time is 3 *years*.

14. In what time will £200 amount to £243. 2·025*s.*, at 5 per cent.? *Ans.* 4 *years.*

15. In what time will £450 amount to £547. 9*s.* 10*d.* 2·0538368*f.*, at 4 per cent.? *Ans.* 5 *years.*

16. In what time will £500 amount to £619. 8*s.* 2*d.* ·38323*f.*, at 5½ per cent.? *Ans.* 4 *years.*

ANNUITIES, OR PENSIONS, IN ARREARS.

NOTE. u represents the annuity, pension, or yearly rent; a, r, t, as before.

A TABLE showing the amount of £1 annuity for any number of years under 31, at 5 and 6 per cent. per annum.

Years.	Rates 5	Rates 6	Years.	Rates 5	Rates 6
1	1·00000	1·00000	16	23·65749	25·67252
2	2·05000	2·06000	17	25·84036	28·21288
3	3·15250	3·18360	18	28·13238	30·90565
4	4·31012	4·37461	19	30·53900	33·75999
5	5·52563	5·63709	20	33·06595	36·78559
6	6·80191	6·97532	21	35·71925	39·99272
7	8·14200	8·39383	22	38·50521	43·39229
8	9·54910	9·89746	23	41·43047	46·99582
9	11·02656	11·49131	24	44·50199	50·81557
10	12·57789	13·18079	25	47·72709	54·86451
11	14·20678	14·97164	26	51·11345	59·15638
12	15·91712	16·86994	27	54·66912	63·70576
13	17·71298	18·88213	28	58·40258	68·52811
14	19·59863	21·01506	29	62·32271	73·63979
15	21·57856	23·27597	30	66·43884	79·05818

NOTE. The above table is made thus:—Take the first year's amount, which is £1, multiply it by r and add 1, for the second year's amount, which also multiply by r and add 1, for the third year's amount, &c.

I. *When u, t, r, are given, to find a.*

RULE. $\frac{(r^t - 1) \times u}{r-1} = a$; or by the table, thus :—

Multiply the amount of £1 for the number of years, and at the rate per cent. given in the question, by the annuity, pension, &c., and it will give the answer.

Examples.

17. What will an annuity of £50 per annum, payable yearly, amount to in 4 years, at 5 per cent. ?

Ans. $1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 - 1 = 59{\cdot}77531250$;

then $\frac{59{\cdot}7753125 \times 50}{1{\cdot}05-1} =$ £215. 10*s.* 1½*d.* Or,

by the table, thus, $4{\cdot}31012 \times 50 =$ £215. 10*s.* 1*d.* 1·76*f.*

18. What will a pension of £45 per annum, payable yearly, amount to in 5 years, at 5 per cent. ? *Ans.* £248. 13*s.* 0*d.* 3·27*f.*

19. If a salary of £40 per annum, to be paid yearly, be forborne 6 years, at 6 per cent., what is the amount ? *Ans.* £279. 0*s.* 3·072*d.*

20. If an annuity of £75 per annum, payable yearly, be omitted to be paid for 10 years, at 6 per cent., what is the amount ?

Ans. £988. 11*s.* 2·22*d.*

II. *When a, r, t, are given, to find u.*

RULE. $\frac{a \times (r-1)}{r^t - 1} = u.$

Examples.

21. What annuity, being forborne 4 years, will amount to £215. 10*s.* 1½*d.* at 5 per cent. ?

Ans. $\frac{215{\cdot}50625 \times {\cdot}05}{1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05-1} =$ £50.

22. What pension being forborne 5 years will amount to £248. 13*s.* 0*d.* 3·27*f.* at 5 per cent. ? *Ans.* £45.

23. What salary being omitted to be paid 6 years will amount to £279. 0*s.* 3·072*d.* at 6 per cent. ? *Ans.* £40.

24. If the payment of an annuity forborne 10 years amount to £988. 11*s.* 2·22*d.* at 6 per cent., what is the annuity ? *Ans.* £75.

III. *When u, a, r, are given, to find t.*

RULE. $\frac{u \times (r-1)}{a} + 1 = r^t$ which being continually divided by r, as far as possible, the number of those divisions will be equal to t.

Examples.

25. In what time will £50 per annum amount to £215. 10*s.* 1½*d.* at 5 per cent. for non-payment?

$$Ans. \frac{215{\cdot}50625 \times {\cdot}05}{50} + 1 = 1{\cdot}21550625$$

which being continually divided by 1·05, the number of those divisions will be = 4, the number of years.

26. In what time will £45 per annum amount to £248. 13*s.* 0*d.* 3·27*f.* allowing 5 per cent. for forbearance of payment? *Ans.* 5 *years.*

27. In what time will £40 per annum amount to £279. 0*s.* 3·072*d.* at 6 per cent.? *Ans.* 6 *years.*

28. In what time will £75 per annum amount to £988. 11*s.* 2·22*d.* allowing 6 per cent. for forbearance of payment? *Ans.* 10 *years.*

PRESENT WORTH OF ANNUITIES, PENSIONS, &c.

A TABLE showing the present worth of £1 annuity for any number of years under 31, discount at 5 and 6 per cent.

Years.	5 Rates	6	Years.	5 Rates	6
1	0·95238	0·94339	16	10·83777	10·10589
2	1·85941	1·83339	17	11·27406	10·47726
3	2·72324	2·67301	18	11·68958	10·82760
4	3·54595	3·46510	19	12·08532	11·15811
5	4·32947	4·21236	20	12·46221	11·46992
6	5·07569	4·91732	21	12·82115	11·76407
7	5·78637	5·58238	22	13·16300	12·04158
8	6·46321	6·20979	23	13·48857	12·30338
9	7·10782	6·80169	24	13·79864	12·55035
10	7·72173	7·36008	25	14·09394	12·78335
11	8·30641	7·88687	26	14·37518	13·00316
12	8·86325	8·38384	27	14·64303	13·21053
13	9·39357	8·85268	28	14·89812	13·40616
14	9·89864	9·29498	29	15·14107	13·59072
15	10·37965	9·71225	30	15·37245	13·76483

NOTE. The above table is thus made: divide £1 by 1·05 = 9·5238, the present worth of the first year, which + 1·05 = ·90703, added to the first year's present worth = 1·85941, the second year's present worth; then, ·90703 + 1·05 and the quotient added to 1·85941 = 2·72324, third year's present worth, &c.

I. *When u, t, r, are given, to find p.*

RULE. $u \times \left(1 - \frac{1}{r^t}\right) \div (r - 1) = p.$

r^t may be taken out of the table at page 134; or, by the above table, thus,

Multiply the present worth of £1 annuity for the time and rate per *cent. given, by the* annuity, pension, &c. it will give the answer.

Examples.

29. What is the present worth of an annuity of £30 per annum, to continue 7 years, at 6 per cent.? *Ans.* £167. 9*s.* 5*d.* ·184*f.*

$$30 \times \left(1 - \frac{1}{1{\cdot}50363}\right) \div {\cdot}06 = 167{\cdot}4716.$$

By the table $5{\cdot}58238 \times 30 = 167{\cdot}4716$.

30. What is the present worth of a pension of £40 per annum, to continue 8 years at 5 per cent.? *Ans.* £258. 10*s.* 6*d.* 3·264*f.*

31. What is the present worth of a salary of £35 to continue 7 years, at 6 per cent.? *Ans.* £195. 7*s.* 7*d.* 3·968*f.*

32. What is the yearly rent of £50, to continue 5 years, worth in ready money, at 5 per cent.? *Ans.* £216. 9*s.* 5*d.* 2·56*f.*

II. *When p, t, r, are given, to find u.*

RULE. $\dfrac{pr^t \times (r-1)}{r^t - 1} = u.$

Examples.

33. If an annuity be purchased for £167. 9*s.* 5*d.* ·184*f.* to be continued 7 years, at 6 per cent., what is the annuity?

Ans. $\dfrac{167{\cdot}4716 \times 1{\cdot}50363 \times {\cdot}06}{{\cdot}50363} = £30.$

34. If the present payment of £258. 10*s.* 6*d.* 3·264*f.* be made for a salary of 8 years to come, at 5 per cent., what is the salary? *Ans.* £40.

35. If the present payment of £195. 7*s.* 7*d.* 3·968*f.* were required for a pension of 7 years to come, at 6 per cent., what is the pension? *Ans.* £35.

36. If the present worth of an annuity, 5 years to come, be £216. 9*s.* 5*d.* 2·56*f.* at 5 per cent., what is that annuity? *Ans.* £50.

III. *When u, p, r, are given, to find t.*

RULE. $\dfrac{u}{u - \times (r-1)} = r^t$ which being continually divided by r, as long as possible, the number of those divisions will be equal to t.

Examples.

37. How long may a lease of £30 yearly rent be had for £167. 9*s.* 5*d.* ·184*f.* allowing 6 per cent. to the purchaser?

$\dfrac{30}{30 - 167{\cdot}4716 \times {\cdot}06} = 1{\cdot}50363$ which being continually divided by 1·06, the number of those divisions will be $= t = 7$ years.

38. If £258. 10*s.* 6*d.* 3·264*f.* is paid down for a lease of annum, at 5 per cent., how long is the lease purchased for?
Ans.

39. If a house is let upon lease for £35 per annum, and th makes present payment of £195. 7*s.* 8*d.*, he being allowed 6 p I demand how long the lease is purchased for? *Ans.*

40. For what time may a lease of £50 per annum be purchas present payment is made of £216. 9*s.* 5*d.* 2·56*f.* at 5 per cent. *Ans.*

ANNUITIES, LEASES, &c. TAKEN IN REVERSI

To find the present worth of annuities, leases, &c. taken in reve

RULE 1. Find the present worth of the annuity, &c. at the given rate, and for the time of its continuance, thus,

$$u \times \left(1 - \frac{1}{r^t}\right) \div (r -$$

2. Change p into a, and find what principal being put to interest will amount to p at the same rate, and for the time to come, before the annuity commences, which will be the present worth of the annuity, &c. thus,

$$\frac{a}{r^t} = p.$$

Examples.

41. What is the present worth of a reversion of a lease of annum, to continue for 6 years, but not to commence till th 2 years, allowing 6 per cent. to the purchaser?
Ans. £175. 1*s.* 1*d*

$$1st, \frac{1}{r^t} = \frac{1}{1{\cdot}41852} = {\cdot}70496.$$

$$2nd, u \times \left(1 - \frac{1}{r^t}\right) \div (r - 1) = \frac{40\,(1 - {\cdot}70496)}{{\cdot}06} = 196{\cdot}693$$

3*rd*, Change p into a, then $\frac{a}{r^t} = \frac{196{\cdot}6933}{1{\cdot}1236}$, (*t*, *after* the chan $= 2) = 175{\cdot}0563 = p$, the present worth.

42. What is the present worth of a reversion of a lease of annum, to continue 7 years, but not to commence till the end o allowing 5 per cent. to the purchaser? *Ans.* £299.

43. There is a lease of a house at £30 per annum, which being for 4 years, and the lessee is desirous to take a lease in for 7 years, to begin when the old lease shall be expired; wh *the present* worth of the said lease in reversion, allowing 5 pe *the purchaser?* Ans. £142. 16*s.* 3*d*

To find the yearly income of an annuity, &c., taken in reversion.

RULE. Find the amount of the present worth, at the given rate, and for the time before the annuity commences. Thus ..... $pr^t = a$.

Change a into p, and find what yearly rent being sold will produce p, at the same rate, and for the time of its continuance, which will be the yearly sum required. Thus ..... $pr^t \times (r-1) \div (r^t - 1) = u$.

Examples.

44. What annuity, to be entered upon 2 years hence, and then to continue 6 years, may be purchased for £175. 1*s.* 1*d.* 2·048*f.*, at 6 per cent.? *Ans.* £40.

1*st*, $pr^t = 175{\cdot}0563 \times 1{\cdot}1236 = 196{\cdot}6933 = a$; then changing this a into p, and taking $t = 6$,

$$pr^t \times (r-1) \div (rt-1) = \frac{196{\cdot}6933 \times 1{\cdot}41852 \times {\cdot}06}{{\cdot}41852} = £40.$$

45. The present worth of a lease of a house is £299. 18*s.* 2·4*d.*, taken in reversion for 7 years, but not to commence till the end of 3 years, allowing 5 per cent. to the purchaser; what is the yearly rent? *Ans.* £60.

46. There is a lease of a house in being for 4 years, and the lessee being minded to take a lease in reversion for 7 years, to begin when the old lease shall be expired, paid down £142. 16*s.* 3*d.* 2·688*f.*; what was the yearly rent of the house, when the lessee was allowed 5 per cent. for present payment? *Ans.* £30.

PURCHASING FREEHOLD OR REAL ESTATES, OR SUCH AS CONTINUE FOR EVER.

I. *When u, r, are given, to find w (the worth).*

RULE. $\frac{u}{r-1} = w$.

Examples.

47. What is the worth of a freehold estate of £50 per annum, allowing 5 per cent. to the buyer?

Ans. $\frac{50}{1{\cdot}05 - 1} = £1000.$

48. What is an estate of £140 per annum, to continue for ever, worth in present money, allowing 4 per cent. to the buyer? *Ans.* £3500.

49. If a freehold estate of £75 yearly rent was to be sold, what is it worth, allowing the buyer 6 per cent.? *Ans.* £1250.

II. *When w, r, are given, to find u.*

RULE. $w \times (r-1) = u$.

Examples.

50. If a freehold estate is bought for £1000, and the allowance of 5 per cent. is made to the buyer, what is the yearly rent?

Ans. $1{\cdot}05 - 1 = {\cdot}05$; *then* $1000 \times {\cdot}05 =$ £50.

51. If an estate be sold for £3500, and 4 per cent. allowed to the buyer, what is the yearly rent? *Ans.* £140.

52. If a freehold estate is bought for £1250 present money, and an allowance of 6 per cent. made to the buyer for the same, what is the yearly rent? *Ans.* £75.

III. *When w, u, are given, to find r.*

RULE. $\dfrac{w + u}{w} = r.$

Examples.

53. If an estate of £50 per annum is bought for £1000, what is the rate per cent.?

Ans. $\dfrac{1000 + 50}{1000} = 1{\cdot}05 = 5$ *per cent.*

54. If a freehold estate of £140 per annum be bought for £3500, what was the rate per cent. allowed? *Ans.* 4 *per cent.*

55. If an estate of £75 per annum is sold for £1250, what is the rate per cent. allowed? *Ans.* 6 *per cent.*

PURCHASING FREEHOLD ESTATES IN REVERSION.

To find the worth of a freehold estate in reversion.

RULE. Find the worth of the yearly rent, thus ..... $\dfrac{u}{r-1} = w.$

Change w into a, and find what principal, being put to interest, will amount to a at the same rate, and for the time to come, before the estate commences, and that will be the worth of the estate in reversion; thus ..... $\dfrac{a}{r^t} = p.$

Examples.

56. If a freehold estate of £50 per annum, to commence 4 years hence, is to be sold, what is it worth, allowing the purchaser 5 per cent. for present payment?

$\dfrac{50}{1{\cdot}05 - 1} = 1000$; *then* $\dfrac{1000}{1{\cdot}2155} =$ £882. 14*s.* 1½*d.*

57. What is an estate of £200, to continue for ever, but not to commence till the end of 2 years, worth in ready money, allowing the purchaser 4 per cent.? *Ans.* £4622. 15*s.* 7·44*d.*

58. What is an estate of £240 per annum worth in ready money, to continue for ever, but not to commence till the end of 3 years, allowance *being made at 6 per cent.*? Ans. £3358. 9*s.* 6*d.* 2·24*f.*

To find the yearly rent of an estate taken in reversion.

RULE. Find the amount of the worth of the estate, at the given rate and time before it commences; thus, $wr^t = a$. Change a into w, and find what yearly rent, being sold, will produce u, at the same rate; thus..... $w(r - 1) = u$. which will be the yearly rent required.

Examples.

59. If a freehold estate, to commence 4 years hence, is sold for £822. 14*s.* 1½*d.*, allowing the purchaser 5 per cent., what is the yearly income?

1*st.* 822·70625 × 1·2155 = 1000; *then* 1000 × ·05 = £50.

60. A freehold estate is bought for £4622. 15*s.* 7·44*d.*, which does not commence till the end of 2 years, the buyer being allowed 4 per cent. for his money; I desire to know the yearly income? *Ans.* £200.

61. There is a freehold estate sold for £3358. 9*s.* 6*d.* 2·24*f.*, but not to commence till the expiration of 3 years, allowing 6 per cent. for present payment; what is the yearly income? *Ans.* £240.

REBATE, OR DISCOUNT

A TABLE showing the present worth of £1, due any number of years, to commence under 31, rebate at 5 and 6 per cent.

Years.	Rates 5	6	Years.	Rates 5	6
1	·952381	·943396	16	·458111	·393647
2	·907030	·889996	17	·436296	·371364
3	·863838	·839619	18	·415520	·350343
4	·822702	·792093	19	·395734	·330513
5	·783526	·747258	20	·376889	·311804
6	·746215	·704960	21	·358942	·294155
7	·710682	·665057	22	·341849	·277505
8	·676839	·627412	23	·325571	·261797
9	·644609	·591898	24	·310067	·246978
10	·613913	·558394	25	·295302	·232998
11	·584679	·526787	26	·281240	·219810
12	·556837	·496969	27	·267848	·207368
13	·530321	·468839	28	·255093	·195630
14	·505068	·442301	29	·242946	·184556
15	·481017	·417265	30	·231377	·174110

NOTE. The above table is thus made:—1 ÷ 1·05 = ·952381, first year's present worth; and ·952381 ÷ 1·05 = ·90703, second year; and ·90703 ÷ 1·05 = ·863838, third year, &c.

I. *When s, t, r, are given, to find p.*

RULE. $\frac{s}{r^t} = p.$

Examples.

1. What is the present worth of £315. 12*s.* 4½*d.* payable 4 years hence, at 6 per cent.?

Ans. 1·06 × 1·06 × 1·06 × 1·06 = 1·26247; *then by the table.*

$$\frac{315\cdot6175}{1\cdot26247} = £250$$

315·6175
·792093
249·9984124275

2. If £344. 14*s.* 9*d.* 1·92*f.* be payable in 7 years' time, what is the present worth, rebate being made at 5 per cent.? *Ans.* £245.

3. There is a debt of £441. 17*s.* 3*d.* 1·92*f.* which is payable 4 years hence, but it is agreed to be paid in present money; what sum must the creditor receive, rebate being made at 6 per cent.? *Ans.* 350.

II. *When p, t, r, are given, to find s.*

RULE. $p \times r^t = s.$

Examples.

4. If a sum of money, due 4 years hence, produce £250 for the present payment, rebate being made at 6 per cent., what was the sum first due? *Ans.* 250 × 1·26247 = £315. 12*s.* 4½*d.*

5. If £245 be received for a debt payable 7 years hence, and an allowance of 5 per cent. to the debtor for present payment, what was the debt? *Ans.* £344. 14*s.* 9*d.* 1·92*f.*

6. There is a sum of money due at the expiration of 4 years, but the creditor agrees to take £350 for present payment, allowing 6 per cent.; what was the debt? *Ans.* £441. 17*s.* 3*d.* 1·92*f.*

II. *When s, p, r, are given, to find t.*

RULE. $\frac{s}{p} = r^t$ which being continually divided by r, as long as possible, the number of those divisions will be equal to t.

Examples.

7. The present payment of £250 is made for a debt of £315. 12*s.* 4½*d.* rebate at 6 per cent.; in what time was the debt payable?

Ans. $\frac{315\cdot6175}{250} = 1\cdot26247$ which being continually divided, those divisions will be equal to 4 = the number of years.

8. A person receives £245 now for a debt of £344. 14*s.* 9*d.* 1·92*f.*, rebate being made at 5 per cent.; I demand in what time the debt was *payable?* *Ans.* 7 *years.*

9. There is a debt of £441. 17*s.* 3*d.* 1·92*f.* due at a certain time to come, but 6 per cent. being allowed to the debtor for the present payment of £350, I desire to know in what time the sum should have been paid without any rebate? *Ans.* 4 *years.*

IV. *When s, p, t, are given, to find r.*

RULE. $\frac{s}{p} = r^t$ which being extracted by the rules of extraction, (the time given in the question showing the power) the root will be equal to r.

Examples.

10. A debt of £315. 12*s.* 4½*d.* is due 4 years hence, but it is agreed to take £250 now, what is the rate per cent. that the rebate is made at?

Ans. $\frac{315{\cdot}6175}{250} = 1{\cdot}26247$; $\sqrt{1{\cdot}26247} = 1{\cdot}06 = 6$ *per cent.*

11. The present worth of £344. 14*s.* 9*d.* 1·22*f.*, payable 7 years hence, is £245, at what rate per cent. is rebate made? *Ans.* 5 *per cent.*

12. There is a debt of £441. 17*s.* 3*d.* 1·92*f.* payable in 4 years time, but it is agreed to take £350 present payment, I desire to know what rate per cent. rebate is made at? *Ans.* 6 *per cent.*

The rules by which the foregoing questions on subjects connected with Interest, Annuities, &c., are worked, are derived from the investigations of Algebra, a department of science which must be studied by all who would acquire a knowledge of the theoretical principles upon which the higher portions of practical arithmetic are based.

PART IV.

DUODECIMALS;

OR, WHAT IS GENERALLY CALLED CROSS MULTIPLICATION, AND SQUARING OF DIMENSIONS BY ARTIFICERS AND WORKMEN.

Rule for Multiplying Duodecimally.

1. Under the Multiplicand write the corresponding denominations of the Multiplier.

2. Multiply each term in the Multiplicand (beginning at the lowest) by the feet in the Multiplier; write each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.

3. In the same manner multiply the Multiplicand by the inches in the Multiplier, and write the result of each term one place more to the right-hand of those in the Multiplicand.

4. Work in the same manner with the seconds in the Multiplier, setting the result of each term two places to the right-hand of those in the Multiplicand, and so on for thirds, fourths, &c.

Examples.

1. Multiply Ft. In. 7 9 by Ft. In. 3 6

Cross Multiplication.	Practice.	Duodecimals.	Decimals.
7 × 9	6 | ½ | 7 9	7 9	7·75
3 × 6	| | 3 6	3 6	3·5
21 0 0 = 7 × 3	23 3	23 3	3875
2 3 0 = 9 × 3	3 10 6	3 10 6	2325
3 6 0 = 7 × 6	27 1 6	27 1 6	27·125
0 4 6 = 9 × 6			
27 1 6			

The answer is 27 ft., 1 twelfth of a foot, and $\frac{6}{12}$ of *1 twelfth*, that is, 6 inches.

	Ft.	In.		Ft.	In.		Ft.	′	″		
2. Multiply	8	5	by	4	7	*Ans.*	38	6	11		
3. Multiply	9	8	by	7	6	*Ans.*	72	6			
4. Multiply	8	1	by	3	5	*Ans.*	27	7	5		
5. Multiply	7	6	by	5	9	*Ans.*	43	1	6		
6. Multiply	4	7	by	3	10	*Ans.*	17	6	10		
7. Multiply	7	5 9′	by	3	5 3′	*Ans.*	25	8	6	2‴	3‴″
8. Multiply	10	4 5	by	7	8 6	*Ans.*	79	11	0	6	6
9. Multiply	75	7	by	9	8	*Ans.*	730	7	8		
10. Multiply	97	8	by	8	9	*Ans.*	854	7			
11. Multiply	57	9	by	9	5	*Ans.*	543	9	9		
12. Multiply	75	9	by	17	7	*Ans.*	1331	11	3		
13. Multiply	87	5	by	35	8	*Ans.*	3117	10	4		
14. Multiply	179	3	by	38	10	*Ans.*	6960	10	6		
15. Multiply	259	2	by	48	11	*Ans.*	12677	6	10		
16. Multiply	257	9	by	39	11	*Ans.*	10288	6	3		
17. Multiply	311	4 7	by	36	7 5	*Ans.*	11402	2	4	11	11
18. Multiply	321	7 3	by	9	3 6	*Ans.*	2988	2	10	4	6

The Application.

Artificer's work is computed by different measures, viz.—

1. Glazing and mason's flatwork by the foot.
2. Painting, plastering, paving, &c. by the yard.
3. Partitioning, flooring, roofing, tiling, &c. by the square of 100 feet.
4. Brickwork, &c. by the rod, or 16½ feet, whose square is 272¼.

MEASURING BY THE FOOT SQUARE, AS GLAZIERS AND MASON'S FLATWORK.

Examples.

19. There is a house with 3 tier of windows, 3 in a tier, the height of the first tier 7 feet 10 inches, the second 6 feet 8 inches, and the third 5 feet 4 inches, the breadth of each is 3 feet 11 inches, what will the glazing come to at 14*d.* per square foot?

Duodecimals.

7	10	} *the*
6	8	} *heights*
5	4	} *added*
19	10	
	3	*windows*
59	6	*in a tier.*
3	11	*in breadth.*
178	6	
54	6	6
233	0	6

Ft.	′	″		
233	0	6	*at* 14*d. per foot.*	
2*d.* ½ 233			at 1*s.*	
38	10		,, 2*d.*	
		½	for 6 *inches.*	
2|0)27|1		10½		
£13	11	10½	*Ans.*	

20. What is the worth of 8 squares of glass, each measuring 4 feet 10 inches long, and 2 feet 11 inches broad, at $4\frac{1}{2}d.$ per foot?
Ans. £2. 2*s.* $3\frac{1}{2}d.$

21. There are 8 windows to be glazed, each measures 1 foot 6 inches wide, and 3 feet in height, how much will they come to at $7\frac{3}{4}d.$ per foot?
Ans. £1. 3*s.* 3*d.*

22. What is the price of a marble slab, whose length is 5 feet 7 inches, and the breadth 1 foot 10 inches, at 6*s.* per foot? *Ans.* £3. 1*s.* 5*d.*

MEASURING BY THE YARD SQUARE, AS PAVIORS, PAINTERS, PLASTERERS, AND JOINERS DO.

NOTE. Divide the square feet by 9, and it will give the number of square yards.

Examples.

23. A room is to be ceiled, whose length is 74 feet 9 inches, and width 11 feet 6 inches, what will it come to at 3*s.* $10\frac{1}{2}d.$ per yard?
Ans. £18 10*s.* $1\frac{1}{2}d.$

24. What will the paving of a court-yard come to, at $4\frac{3}{4}d.$ per yard, the length being 58 feet 6 inches, and breadth 54 feet 9 inches?
Ans. £7. 0*s.* $10\frac{1}{2}d.$

25. A room painted, 97 feet 8 inches about, and 9 feet 10 inches high, what does it come to at 2*s.* $8\frac{3}{4}d.$ per yard? *Ans.* £14. 11*s.* $2\frac{1}{4}d.$

26. What is the content of a piece of wainscoting in yards square, that is 8 feet 3 inches long, and 6 feet 6 inches broad, and what will it come to at 6*s.* $7\frac{1}{2}d.$ per yard? *Ans.* £1. 19*s.* $5\frac{1}{2}d.$

27. What will the paving a court-yard come to, at 3*s.* 2*d.* per yard, if the length be 27 feet 10 inches, and the breadth 14 feet 9 inches?
Ans. £7. 4*s.* $5\frac{1}{2}d.$

28. A person has paved a court-yard 42 feet 9 inches in front, and 68 feet 6 inches in depth, and in this he laid a footway the depth of the court, of 5 feet 6 inches in breadth: the footway is laid with purbeck stone at 3*s.* 6*d.* per yard, and the rest with pebbles at 3*s.* per yard, what will the whole come to? *Ans.* £49. 17*s.* $0\frac{1}{2}d.$

29. What will the plastering of a ceiling, at 10*d.* per yard, come to, supposing the length 21 feet 8 inches, and the breadth 14 feet 10 inches?
Ans. £1. 9*s.* 9*d.*

30. What will the wainscoting a room come to, at 6*s.* per square yard, supposing the height of the room (taking in the cornice and moulding) is 12 feet 6 inches, and the compass 83 feet 8 inches, the three window shutters each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the shutters and door being worked on both sides, is reckoned work and half work? *Ans.* £36. 12*s.* $2\frac{1}{2}d.$

MEASURING BY THE SQUARE OF 100 FEET, AS FLOORING, PARTITIONING, ROOFING, TILING, &C.

Examples.

31. In 173 feet 10 inches in length, and 10 feet 7 inches in height of *partitioning, how many* squares? *Ans.* 18 squares, 39 feet, 8 10".

32. A house of 3 stories, besides the ground floor, was to be floored at £6. 10*s.* per square, and the house measures 20 feet 8 inches by 16 feet 9 inches: there are seven fire-places, whose measures are two of 6 feet by 4 feet 6 inches each, two of 6 feet by 5 feet 4 inches each, and two of 5 feet 8 inches by 4 feet 8 inches, and the seventh of 5 feet 2 inches by 4 feet, and the well-hole for the stairs is 10 feet 6 inches by 8 feet 9 inches, what will the whole come to? *Ans.* £53. 13*s.* 3½*d.*

33. If a house measures within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of a true pitch, what will it come to roofing, at 10*s.* 6*d.* per square? *Ans.* £12. 12*s.* 11¾*d.*

NOTE. In tiling, roofing, and slating, it is customary to reckon the flat, and half of any building within the wall, to be the measure of the roof of that building, when the said roof is of a true pitch, *i.e.*, when the rafters are three-fourths of the breadth of the building; but if the roof is more or less than the true pitch, they measure from one side to the other with a rod or string.

34. What will the tiling of a barn cost, at 25*s.* 6*d.* per square, the length being 43 feet 10 inches, and breadth 27 feet 5 inches on the flat, the eave boards projecting 16 inches on each side? *Ans.* £24. 9*s.* 5¾*d.*

MEASURING BY THE ROD.

NOTE. Bricklayers always value their work at the rate of a brick and a half thick; and if the thickness of the wall is more or less, it must be reduced to that thickness by this

RULE. Multiply the area of the wall by the number of half-bricks the thickness of the wall is of; the product, divided by 3, gives the reduced area, which, divided by 272¼, will give the rods of brickwork.

Examples.

35. If the area of a wall be 4085 feet, and the thickness two bricks and a half, how many rods doth it contain? *Ans.* 25 *rods*, 2 *square feet.*

36. If a garden wall be 254 feet round, and 12 feet 7 inches high, and 3 bricks thick, how many rods doth it contain?

Ans. 23 *rods* 130 *feet.*

37. How many square rods are there in a wall 62½ feet long, 14 feet 8 inches high, and 2½ bricks thick? *Ans.* 5 *rods* 166 *feet.*

38. The side walls of a house are 28 feet 10 inches in length, and the height of the roof from the ground 53 feet 8 inches, and the gable (or triangular part at top) rises 42 course of bricks, reckoning 4 course to a foot. Now, 20 feet high is at 2½ bricks thick, 20 feet more, at 2 bricks thick, 15 feet 8 inches more, at 1½ brick thick, and the gable at 1 brick thick, what will the whole work come to, at £5. 16*s.* per rod? *Ans.* £48. 12*s.* 7*d.*

PART V.

A COLLECTION OF QUESTIONS,

SET DOWN PROMISCUOUSLY, FOR FURTHER EXERCISE IN THE FOREGOING RULES.

1. Write down two millions, five hundred and two thousand, two hundred and five.

2. What is the value of 14 barrels of soap, at $4\frac{1}{2}d.$ per lb., each barrel containing 254 lb.? *Ans.* £66. 13*s.* 6*d.*

3. If £100 principal gain £5 interest in 12 months, what principal will gain £20 in 8 months? *Ans.* £600.

4. What number is that from which, if the square of 14 be deducted, and to the remainder the square of 12 be added, the sum will be 250? *Ans.* 302.

5. A and B trade together; A put in £320 for 5 months, B 460 for 3 months, and they gained £100; what must each man receive? *Ans.* A £53. 13*s.* 9*d.* $\frac{135}{148}$, and B £46. 6*s.* 2*d.* $\frac{14}{148}$.

6. How many yards of cloth, at 17*s.* 6*d.* per yard, can I have for 13 cwt. 2 qrs. of wool, at 14*d.* per lb.? *Ans.* 100 *yards* $3\frac{1}{2}$ *qrs.*

7. What number, added to the cube of 21, will make the sum equal to 113 times 147? *Ans.* 7350.

8. If I buy 1000 ells Flemish of linen for £90, what may I sell it at per ell in London to gain £10 by the whole? *Ans.* 3*s.* 4*d. per ell.*

9. A has 648 yards of cloth, at 14*s.* per yard ready money, but in barter will have 16*s.* B has wine at £42 per tun ready money; the question is, how much wine must be given for the cloth, and what is the price of a tun of wine in barter? *Ans.* £48 *the tun, and* 10 *tun* 3 *hhds.* 12 *gal. of wine must be given for the cloth.*

10. A jeweller sold jewels to the value of £1200, for which he received in part 876 French pistoles, at 16*s.* 6*d.* each, what sum remains unpaid? *Ans.* £477. 6*s.*

11. An oilman bought 417 cwt. 1 qr. 15 lb. gross weight of train oil, *tare 20 lb. per* 112 lb., how many neat gallons were there, allowing $7\frac{1}{2}$ *lb. to a gallon?* Ans. 5120 gallons.

12. I bought threescore pieces of Holland for three times as many pounds, and sold them again for four times as much: but if they had cost me as much as I sold them for, what should I have sold them for to gain after the same rate? *Ans.* £320.

13. What number, taken from the square of 54, will leave nineteen times 46? *Ans.* 2042.

14. If I buy a yard of cloth for 14*s.* 6*d.*, and sell it for 16*s.* 9*d.*, what do I gain per cent.? *Ans.* £15. 10*s.* 4*d.* $\frac{4}{11}$.

15. Bought 27 bags of ginger, each weighing gross 84$\frac{3}{4}$ lb., tare 1$\frac{3}{8}$ lb. per bag, trett 4 lb. per 104 lb., what do they come to at 8$\frac{1}{2}$*d.* per lb.? *Ans.* £76. 13*s.* 2$\frac{1}{2}$*d.*

16. If $\frac{3}{8}$ of an ounce cost $\frac{7}{8}$ of a shilling, what will $\frac{5}{8}$ of a lb. cost? *Ans.* 17*s.* 6*d.*

17. If $\frac{5}{8}$ of a gallon cost $\frac{5}{8}$ of a £, what will $\frac{5}{8}$ of a tun cost? *Ans.* £105.

18. A young man received £210, which was $\frac{3}{5}$ of his eldest brother's portion; now three times the eldest brother's portion was half of the father's estate, I demand how much the estate was? *Ans.* £1890.

19. If the salary of an officer be £48 per annum, what must he receive for 232 days? *Ans.* £30. 10*s.* 2$\frac{1}{2}$*d.*

20. A gentleman spends, one day with another, £1. 7*s.* 10$\frac{1}{2}$*d.*, and at the year's end layeth up £340, what is his yearly income? *Ans.* £848. 14*s.* 4$\frac{1}{2}$*d.*

21. A lady's fortune consisted of a cabinet worth £200, containing 16 drawers, each having two partitions, each of which contained £37 and two crowns, pray what was her portion? *Ans.* £1400.

22. A has 13 fother of lead to send abroad, each being 19$\frac{1}{2}$ times 112 lb. B has 39 casks of tin, each 388 lb., how many ounces difference is there in the weight of these commodities? *Ans.* 212160 *oz.*

23. A captain and 160 sailors took a prize worth £1360, of which the captain had $\frac{1}{4}$ for his share, and the rest was equally divided among the sailors, what was each man's part? *Ans. the captain had* £272, *and each sailor* £6. 16*s.*

24. What number is that, to which if you add 7$\frac{2}{3}$ the whole will be 12$\frac{1}{4}$? *Ans.* 4$\frac{7}{12}$.

25. An usurer put out £75 for 12 months, and received for principal and interest £81; I demand at what rate per cent. he received interest? *Ans.* 8 *per cent.*

26. What will £956 amount to in 7$\frac{1}{2}$ years, at 5 per cent. simple interest? *Ans.* £1314. 10*s.*

27. At what rate per cent. will £956 amount to £1314. 10*s.* in 7$\frac{1}{2}$ years, at simple interest? *Ans.* 5 *per cent.*

28. If for £1. 4*s.* I have 1200 lb. weight carried 35 miles, how many lb. weight can I have carried 24 miles for the same money? *Ans.* 1800 *lb.*

29. If 8 cannons in one day spend 48 barrels of powder, I demand how many barrels 24 cannons will spend in 22 days? *Ans.* 3168.

30. What number is that, which being multiplied by $\frac{2}{3}$, will produce $\frac{1}{4}$? *Ans.* $\frac{3}{8}$.

31. A has 24 cows worth 72*s.* each, and B seven horses worth £13 apiece, how much will make good the difference in case they interchange their said drove of cattle? Ans. £4. 12*s.*

32. A man dies and leaves £120 to be given to three persons, viz., A, B, and C; to A a share unknown; B twice as much as A, and C as much as A and B; what was the share of each?

Ans. A £20, B £40, *and* C £60.

33. A person dying left his widow £1780, and £1250 to each of his four children; he had been 25¼ years in trade, and had cleared (at an average) £126 a year, what had he to begin with? *Ans.* £3567.

34. There is a sum of £1000 to be divided among 3 men in such manner, that if A has £3, B shall have £5, and C £8; how much must each man have? *Ans.* A £187. 10*s.*, B £312. 10*s.*, *and* C £500.

35. A piece of wainscot is 8 feet 6½ inches long, and 2 feet 9¾ inches broad, what is the superficial content? *Ans.* 24 *feet*, 0′ 3″ 4‴ 6⁗.

36. How many changes may be rung on 6 bells? *Ans.* 720.

37. A merchant at Amsterdam is indebted to another in London £642, and would pay it in Spanish guilders, at 2*s.* per piece, how many must the English merchant receive? *Ans.* 6420.

38. If 360 men be in garrison, and have provision for 6 months, but hearing of no relief at the end of 5 months, how many men must depart, that the provisions may last so much the longer?

Ans. 288 *men.*

39. The less of two numbers is 187, their difference 34, the square of the product is required? *Ans.* 1707920929.

40. A butcher sends his man with £216 to a fair, to buy cattle: oxen at £11, cows at 40*s.*, colts at £1. 5*s.*, and hogs at £1. 15*s.* per piece, and of each a like number, how many of each sort did he buy?

Ans. 13 *of each sort, and* £8 *over.*

41. What number, added to 11$\frac{5}{7}$, will produce 36$\frac{337}{616}$? *Ans.* 24$\frac{513}{616}$.

42. What number, multiplied by $\frac{3}{7}$, will produce 11$\frac{2}{17}$? *Ans.* 26$\frac{41}{51}$.

43. A man had 12 sons; the youngest was 3 years old, and the eldest 58; they increased in arithmetical progression; what was the common difference of their ages? *Ans.* 5 *years.*

44. What is the value of 179 hogsheads of tobacco, each weighing 13 cwt., at £2. 7*s.* 1*d.* per cwt.? *Ans.* £5478. 2*s.* 11*d.*

45. My factor sends me word he has bought goods to the value of £500. 13*s.* 6*d.* upon my account, what will his commission come to, at 3½ per cent.? *Ans.* £17. 10*s.* 5½*d.* $\frac{1}{2}\frac{1}{2}f.$

46. Miss Kitty told her sister Charlotte, whose father had before left them £13200 a-piece, that their grandmother by will had raised her fortune to £15000, and had made her own £20000, what did the old lady leave them? *Ans.* £8600.

47. A snail in getting up a May-pole only 20 feet high, was observed to climb 8 feet every day, but every night he came down again 4 feet; in what time by this method did he reach the top of the pole?

Ans. 4 *days.*

48. If the $\frac{1}{3}$ of 6 were 3, what would $\frac{1}{4}$ of 20 be? *Ans.* 7½.

49. What is the difference between 14676, and the fourth of itself?

Ans. 11007.

50. There is in three bags the sum of £1468, viz., in the first bag *£461, in the second £581*, what was in the third bag? *Ans.* £426.

51. What is the decimal of 3 qrs. 14 lb. of an cwt. Ans. ·875 cwt.

52. How many lb. of sugar, at $4\frac{1}{2}d.$ per lb., must be given in barter for 60 gross of incle, at 8*s.* 8*d.* per gross? *Ans.* $1386\frac{2}{3}$.

53. If I buy yarn for 9*d.* the lb., and sell it again for $13\frac{1}{2}d.$ per lb., what is the gain per cent.? *Ans.* £50.

54. A tobacconist would mix 20 lb. of tobacco at 9*d.* per lb. with 60 lb. at 12*d.* per lb., 40 lb. at 18*d.* per lb., and with 12 lb. at 2*s.* per lb.; what is a lb. of this mixture worth? *Ans.* 1*s.* $2\frac{1}{2}d.$ $\frac{8}{11}$.

55. What is the value of 14 barrels of soap, at $4\frac{1}{2}d.$ per lb., each barrel containing 254 lb.? *Ans.* £66. 13*s.* 6*d.*

56. Two persons, A and B, owe several debts; the lesser debt, being that of A, is £2173, the difference is £371; what is the debt of B? *Ans.* £2544.

57. What is the difference between twice eight and twenty and twice twenty-eight? as also between twice five and fifty and twice fifty-five? *Ans.* 20 *and* 50.

58. What number, taken from the square of 54, will leave 19 times 46? *Ans.* 2042.

59. A schoolmaster being asked how many scholars he had, said, If I had as many more, half as many, and one quarter as many, I should have 99; how many had he? *Ans.* 36.

60. An elderly lady being asked how old she was, to avoid a direct answer said, I have 9 children, and there are 3 years between the birth of each of them; the eldest was born when I was 19 years old, which is now exactly the age of the youngest; how old was the lady? *Ans.* 62.

61. What number is that which, being added to 168, makes the sum to be 706? *Ans.* 538.

62. Multiply $87654321\frac{1}{2}$ by $26\frac{2}{3}$. *Ans.* $2337448573\frac{1}{3}$.

63. If when wheat is 4*s.* the bushel, the 20-penny loaf weighs 18 lb., what must the said 20-penny loaf weigh when wheat is 6*s.* the bushel? *Ans.* 12 *lb.*

64. Whereas a noble and a mark just 15 yards did buy,
How many ells of the same cloth for £50 had I?* *Ans.* 600.

65. A broker bought for his principal, in the year 1720, £400 capital stock in the South Sea, at £650 per cent., and sold it again when it was worth but £130 per cent.; how much was lost in the whole? *Ans.* £2080.

66. What number, added to the 43rd part of 4429, will make the sum 240? *Ans.* 137.

67. What number, deducted from the 26th part of 2262, will leave the 87th part of the same? *Ans.* 61.

68. A gentleman went to sea at 17 years of age; 8 years after that he had a son born, who lived 46 years, and died before his father; after whom the father lived twice 20 years, and then died also; what was the age of the father when he died? *Ans.* 111.

69. C has candles at 6*s.* per dozen ready money, but in barter will have 6*s.* 6*d.* per dozen; D has cotton at 9*d.* per lb. ready money, I

*The value of a noble was 6s. 8d.; of a mark, 13s. 4d.

demand what price the cotton must be at in barter; also how much cotton must be bartered for 100 dozen of candles?

Ans. The cotton at $9\frac{3}{4}$*d. per lb., and* 7 *cwt.* 0 *qr.* 16 *lb. of cotton must be given for* 100 *dozen of candles.*

70. The sum of two numbers is 360, the less 114, what is their difference, product, and larger quote?

Ans. 132 *difference,* 28044 *product,* $7\frac{8}{19}$.

71. A brigade of horse, consisting of 384 men, is to be formed into a square body, having 32 men in front, how many ranks will there be?

Ans. 12.

72. If a clerk's salary be £73 a year, what is that per day?

Ans. 4*s.*

73. B has an estate of £53 per annum, and pays 5*s.* 10*d.* to the subsidy; what must C pay, whose estate is worth £100 per annum?

Ans. 11*s.* $0\frac{4}{53}$*d.*

74. If I buy 100 yards of ribbon at 3 yards for a shilling, and 100 more at 2 yards for a shilling, and sell it at the rate of 5 yards for 2 shillings, whether do I get or lose, and how much?

Ans. lose 3*s.* 4*d.*

75. What is the value of $\frac{4}{5}$ of 20*s.*? *Ans.* 12*s.* 6*d.*

76. What number is that, from which if you take $\frac{2}{3}$ the remainder will be $\frac{1}{8}$? *Ans.* $\frac{24}{40}$.

77. My purse and money, said Dick, are worth 12*s.* 8*d.*, but the money is worth 7 of the purse; pray what is the sum therein?

Ans. 11*s.* 1*d.*

78. What number is that which makes 9 to be the $\frac{3}{5}$ of it?

Ans. $13\frac{1}{3}$.

79. A maltster has several sorts of malt, one at 4*s.* 6*d.*, another at 4*s.*, and a third at 3*s.* 6*d.* a bushel; to mix an equal quantity of each, what must be the price of a bushel? *Ans.* 4*s.*

80. A farmer is willing to make a mixture of rye at 4*s.* a bushel, barley at 3*s.*, and oats at 2*s.*; how much must he take of each to sell it at 2*s.* 6*d.* the bushel? *Ans.* 6 *of rye,* 6 *of barley, and* 24 *of oats.*

81. If $\frac{3}{8}$ of a ship be worth £3740, what is the worth of the whole?

Ans. £9973. 6*s.* 8*d.*

82. A person said he had 20 children, and that it happened there was a year and a half between each of their ages; his eldest son was born when he was 24 years old, and the age of his youngest is 21; what was the father's age? *Ans.* $73\frac{1}{2}$ *years.*

83. Bought a cask of wine for £62. 8*s.*; how many gallons were in the same, when a gallon was valued at 5*s.* 4*d.*? *Ans.* 234.

84. B owes C £296. 17*s.*, but he compounds for 7*s.* 6*d.* in the pound, what must C receive for his debt? *Ans.* £111. 6*s.* $4\frac{1}{2}$*d.*

85. How many dozen of stockings, at 11 groats per pair, may I buy for £190. 12*s.*? *Ans.* 86 *dozen* 7 *pair* $\frac{7}{11}$.

86. A sheepfold was robbed 3 nights successively; the first night half the sheep were stolen, and half a sheep more; the second night half the remainder were lost, and half a sheep more; the last night *they took half that* were left, and half a sheep more; by which time *they were reduced to* 20; how many were they at first? Ans. 161.

87. The Spectator's club of fat people, though it consisted but of 15 persons, is said to have weighed no less than 3 tons; how much at an equality was that per man? *Ans.* 4 *cwt.*

88. A reckless young fellow in a small time got the better of $\frac{1}{5}$ of his fortune; by advice of his friends he gave £2200 for an Exempt's place in the Guards; his profusion continued till he had no more than 880 guineas left, which he found by computation was $\frac{3}{40}$ part of his money after the commission was bought; pray what was his fortune at first? *Ans.* £10450.

89. B owes C £395. 18*s.*, but compounds the whole debt for £100. 12*s.*, what is that in the pound? *Ans.* 5*s.* $0\frac{3}{4}$*d.* $\frac{3783}{5050}$.

90. How many dollars, at 4*s.* 4*d.* each, must be given for 360 guilders, at 2*s.* 2*d.* each? *Ans.* 180.

91. Four men have a sum of money to be divided amongst them, in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{5}$, and the fourth the remainder, which is 28; what is the sum? *Ans.* 112.

92. What is the amount of £1000 for $5\frac{1}{2}$ years, at $4\frac{3}{4}$ per cent., simple interest? *Ans.* £261. 5*s.*

93. Sold goods to the value of £700, for 4 and 8 months; what is the (banker's) present worth, at 5 per cent. simple interest? *Ans.* £682. 10*s.*

94. A room 30 feet long and 18 feet wide, is to be covered with painted cloth; how many yards of $\frac{3}{4}$ wide will cover it? *Ans.* 80 *yards.*

95. There are two numbers, the one 48, the other twice as much; what is the difference between their sum and difference? *Ans.* £96.

96. Hetty told her brother George, that though her fortune on her marriage took £19312 out of her family, it was but $\frac{2}{3}$ of two years' rent of his yearly income; pray what was that? *Ans.* £16093. 6*s.* 8*d. a-year.*

97. There are two numbers, the one 25, the other the square of 25; I demand the square root of the sum of their squares? *Ans.* 625·4998.

98. Says B to C, If I had 4 of your sheep I should have as many as you; and says C to B, If I had 4 of yours I should have twice as many as you; how many had each? *Ans.* B 20, C 28.

99. B, C, and D trading together gained £120, which is to be shared according to each man's stock; B put in £140, C £300, and D £160; what is each man's share? *Ans.* B £28, C £60, D £32.

100. A gentleman having 50*s.* to pay amongst his labourers for a day's work, would give to every boy 6*d.*, to every woman 8*d.*, and to every man 16*d.*; the number of boys, women, and men was the same; I demand the number of each? *Ans.* 20 *of each.*

101. There are 3 numbers, 17, 19, and 48; I demand the difference between the sum of the squares of the first and last, and the cube of the middle one? *Ans.* 4266.

102. A stone that measures 4 feet 6 inches long, 2 feet 9 inches broad, and 3 feet 4 inches deep, how many solid feet doth it contain? *Ans.* $41\frac{1}{4}$ *solid feet.*

103. What does the whole pay of a man-of-war's crew of 640 sailors amount to for 32 months' service, each man's pay being 22*s.* 6*d.* per month? Ans. 23040.

104. If I have an estate of £470 per annum, what may I expend daily, and yet lay up £130 per annum? *Ans.* 18*s.* 7½*d.*

105. What number is that, which being divided by 19, the quotient will be 72? *Ans.* 1368.

106. Reduce 13½ bushels of coals to the fraction of a chaldron. *Ans.* $\frac{3}{8}$.

107. Bought 28 qrs. 2 bushels of wheat, at 4*s.* 6*d.* per bushel, what does it come to? *Ans.* £50. 17*s.*

108. How many pounds of coffee, at 5*s.* 9*d.* per lb., is equal in value to 426 lb. of tea, at 13*s.* 4*d.* per lb.? *Ans.* £987. $\frac{12}{23}$.

109. What is the value of 27 dozen 10 lb. of candles, at 5*d.* per lb.? *Ans.* £6. 19*s.* 2*d.*

110. A traveller would change 500 French crowns, at 4*s.* 6*d.* per crown, into sterling money, but he must pay a halfpenny per crown for change; how much must he receive? *Ans.* £111. 9*s.* 2*d.*

111. There are two numbers, the one 63 and the other half as much. I demand the product of their squares, and the difference of their product and sum?

Ans. product of their squares 3938240·25, *difference* 1890.

112. B and C traded together, and gained £100; B put in £640; C put in so much that he might receive £60 of the gain; I demand how much C put in? *Ans.* £960.

113. Of what principal sum did £20 interest arise in one year, at the rate of 5 per cent. per annum? *Ans.* £400.

114. Having bought 40 yards of cloth, at 8*s.* per yard, and 70 yards at 12*s.*, what is the value of both pieces? *Ans.* £58.

115. Two men depart both from one place, the one goes north, the other south; the one goes 7 miles and the other 11 miles a day; how far are they distant at the twelfth day of their departure?

Ans. 216 *miles.*

116. In 672 Spanish guilders of 2*s.* each, how many French pistoles, at 17*s.* 6*d.* per piece? *Ans.* 76$\frac{4}{7}$.

117. In 7 cheeses, each weighing 1 cwt. 2 qrs. 5 lb., how many allowances for seamen may be cut, each weighing 5 oz. 7 drams?

Ans. 3563$\frac{24}{37}$.

118. If 48 taken from 120 leaves 72, and 72 taken from 91 leaves 19, and 7 taken from thence leaves 12, what number is that, out of which, when you have taken 48, 72, 19, and 7, leaves 12? *Ans.* 158.

119. A farmer, ignorant of numbers, ordered £500 to be divided among his 5 sons thus: give A, says he, $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{1}{6}$, and E $\frac{1}{7}$ part; divide this equitably among them, according to the father's intention.

Ans. A £152$\frac{282}{455}$, B £114$\frac{174}{455}$, C £91$\frac{231}{455}$, D £76$\frac{118}{455}$, E £65$\frac{144}{455}$.

120. If 12 oxen will eat 3½ acres of grass in 4 weeks, and 21 oxen will eat 10 acres in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being allowed to grow uniformly? *Ans.* 36.

121. A lady was asked her age, who replied thus:—If you multiply my age by three, and take two-sevenths of thrice the product, the square root of two-ninths of the result will be four: required, her age?

Ans. 28 *years.*

A TABLE for finding the Interest of any sum of money for any number of months, weeks, or days, at any rate per cent.

Year.	Calendar Month.			Week.			Day.		
£.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1	0	1	8	0	0	4⅓	0	0	0⅓
2	0	3	4	0	0	9	0	0	1¼
3	0	5	0	0	1	1¾	0	0	2
4	0	6	8	0	1	6½	0	0	2⅔
5	0	8	4	0	1	11	0	0	3¼
6	0	10	0	0	2	3¾	0	0	4
7	0	11	8	0	2	8¼	0	0	4⅔
8	0	13	4	0	3	1	0	0	5¼
9	0	15	0	0	3	5½	0	0	6
10	0	16	8	0	3	10¼	0	0	6½
20	1	13	4	0	7	8½	0	1	1½
30	2	10	0	0	11	6⅔	0	1	7¾
40	3	6	8	0	15	4⅓	0	2	2¼
50	4	3	4	0	19	2¾	0	2	9
60	5	0	0	1	3	1	0	3	3½
70	5	16	8	1	6	11	0	3	10
80	6	13	4	1	10	9¼	0	4	4½
90	7	10	0	1	14	7½	0	4	11½
100	8	6	8	1	18	5½	0	5	5¾
200	16	13	4	3	16	11	0	10	11½
300	25	0	0	5	15	4½	0	16	5¼
400	33	6	8	7	13	10	1	1	11
500	41	13	4	9	12	3½	1	7	4¾
600	50	0	0	11	10	9	1	12	10½
700	58	6	8	13	9	2¾	1	18	4¼
800	66	13	4	15	7	8½	2	3	10
900	75	0	0	17	6	1¾	2	9	3¾
1000	83	6	8	19	4	7¼	2	14	9½
2000	166	13	4	38	9	2¾	5	9	7
3000	250	0	0	57	13	10	8	4	4½
4000	333	6	8	76	18	5½	10	19	2
5000	416	13	4	96	3	0¾	13	13	11½
6000	500	0	0	115	7	8½	16	8	9
7000	583	6	8	134	12	3½	19	3	6½
8000	666	13	4	153	16	11	21	18	4½
9000	750	0	0	173	1	6¼	24	13	1¾
10000	833	6	8	192	6	1¾	27	7	11¼
20000	1666	13	4	384	12	3½	54	15	10½
30000	2500	0	0	576	18	5¼	82	3	10

RULE. Multiply the principal by the rate per cent., and by the number of months, weeks, or days which are required; cut off two figures on the *right-hand side of* the product, and collect from the table the several sums against the different numbers, which when added will make

the number remaining. Add the several sums together, it will give the interest required.

N.B. For every 10 that is cut off in months, add 2*d.*; for every 10 cut off in weeks, add a halfpenny; and for every 40 in the days, 1 farthing.

Examples.

1. What is the interest of £2467. 10*s.* for 10 months, at 4 per cent. per annum?

£.	s.
2467	10
	4
9870	0
10	0
987|00	

		£.	s.	d.
900	=	75	0	0
80	=	6	13	4
7	=	0	11	8
987	=	82	5	0

2. What is the interest of £2467. 10*s.* for 12 weeks, at 5 per cent.?

£.	s.
2467	10
	5
12337	10
	12
1480|50	0

		£.	s.	d.
1000	=	19	4	7½
400	=	7	13	10
80	=	1	10	9½
50	=	0	0	2½
1480|50	=	28	9	5

3. What is the interest of £2467. 10*s.* for 50 days, at 6 per cent.?

£.	s.
2467	10
	6
14805	0
50	0
7402|50	0

		£.	s.	d.
7000	=	19	3	6½
400	=	1	1	11
2	=	0	0	1½
50	=	0	0	0½
7402|50	=	20	5	7

To find what an estate, from £1000 to £60000 per annum, will come to for one day.

RULE. Collect the annual rent or income from the table for one year, against which take the several sums for one day; add them together, it will give the answer.

An estate of £376 per annum, what is that per day?

		£.	s.	d.
300	=	0	16	5½
70	=	0	3	10
6	=	0	0	4
376	=	1	0	7½

To find the amount of any income, salary, or servant's wages, for any number of months, weeks, or days.

RULE. Multiply the yearly income or salary by the number of months, weeks, or days, and collect the product from the table.

What will £270 per annum come to for 11 months, for 3 weeks, and for 6 days?

For 11 *Months.*

			£.	s.	d.
270	2000	=	166	13	4
11	900	=	75	0	0
	70	=	5	16	8
2970					
	2970	=	247	10	0

For 3 *Weeks.*

			£.	s.	d.
270	800	=	15	7	$8\frac{1}{4}$
3	10	=	0	3	$10\frac{1}{4}$
810		=	15	11	$6\frac{1}{2}$

For 6 *Days.*

			£.	s.	d.
270	1000	=	2	14	$9\frac{1}{2}$
6	600	=	1	12	$10\frac{1}{2}$
	20	=	0	1	$1\frac{1}{4}$
1620					
	1620	=	4	8	$9\frac{1}{4}$

For the whole time.

£.	s.	d.
247	10	0
15	11	$6\frac{1}{2}$
4	8	$9\frac{1}{4}$
257	10	$3\frac{3}{4}$

A TABLE, showing the number of days from any day in the month, to the same day in any other month, through the year.

From	To	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	Jan.	365	31	59	90	120	151	181	212	243	273	304	334
	Feb.	334	365	28	59	89	120	150	181	212	242	273	303
	Mar.	306	337	365	31	61	92	122	153	184	214	245	275
	April	275	306	334	365	30	61	91	122	153	183	214	244
	May	245	276	304	335	365	31	61	92	123	153	184	214
	June	214	245	273	304	334	365	30	61	91	122	153	183
	July	184	215	243	274	304	335	365	31	62	92	123	153
	Aug.	153	184	212	243	273	304	334	365	31	61	92	122
	Sept.	122	153	181	212	242	273	303	334	365	30	61	91
	Oct.	92	123	151	182	212	243	273	304	335	365	31	61
	Nov.	61	92	120	151	181	212	242	273	304	334	365	30
	Dec.	31	62	90	121	151	182	212	243	274	304	335	365

If the end of February be included between the two points of time, a day must be added in leap year.

[NOTE. *The mathematical principles upon which the rules in the foregoing Work are founded, more especially those concerning Compound Interest, Annuities, &c., will be demonstrated in the first volume of the* SCHOOL COURSE OF MATHEMATICS, *announced for publication at the end of the Preface.*]

APPENDIX.

ON THE DECIMAL COINAGE.

A CHANGE in the monetary system of this kingdom has for some time been under the contemplation of government. The object proposed by the change is to simplify the keeping of accounts, and to facilitate all calculations in which money is concerned. The arithmetical notation used at present in all civilised countries—as far as merely abstract numbers are concerned—is perfect, if what is unsusceptible of improvement may be so called:—it is the *decimal system* of notation. It is thus designated, because any number in this system being written down, the second figure, counting from right to left, represents so many *tens* of the units denoted by the preceding or first figure; the third figure, so many *tens* of the units implied in the second figure, and so on; or, proceeding from the unit's figure in the contrary direction (as in what are more emphatically called *decimals*), we in like manner denote *tenths*, *tenths* of these tenths, and so on.

Every one at all acquainted with arithmetic, must perceive the advantages of this system of numeration over every other; a person has only to reflect for a moment upon the simple character of the operations of addition, subtraction, multiplication, and division, to be enabled to form some estimate of the complication that would be introduced if the *carryings* were other than what they are—so many *tens*.

Such complexity is, however, actually introduced, to a considerable extent, into all our operations with *concrete* quantities, inasmuch as the units of the several denominations have not a tenfold rate of increase, nor, in general, any uniform rate of increase at all: thus, in passing from the farthing, the lowest money-denomination, to the next denomination, the penny, the increase is four fold; from that to the shilling, it is twelve fold; and from that to the pound it is twenty fold. In weights and measures, matters are still worse, because in these the concrete denominations are more numerous.

It is obvious that, leaving the practical inconvenience of this want of

uniformity in the rate of increase altogether out of consideration, nothing can be more unsystematic and capricious than to be thus changing our scale of notation every time we proceed from one concrete denomination to another. Why should our concrete arithmetic differ at all, as to its notational gradations, from abstract arithmetic, seeing that the latter combines all the advantages of uniformity of system with practical facility?

Many proposals have recently been made to remove this incongruity, as respects our money calculations; that is, to introduce into such calculations the *decimal system*, and so to do away with the distinction between addition, subtraction, &c., of *money*, and the like operations with abstract numbers. It is not my purpose here to offer any opinion in reference to the different plans that have been proposed; but I think the following suggestions are worthy of consideration.

And first, I would remark that whatever value be given to our highest money-unit, which at present is the *pound*, or 240 *pence*, and by whatever names we agree to call the decimal sub-divisions of it, the facilities of calculation remain pretty nearly the same: as far as the inferior denominations are concerned, they are indeed *exactly* the same; the only difference being, that the *smaller* the value of the highest money-unit, the *larger* will be the number expressing what we now call *pounds*.

As, therefore, the value of the highest money-unit is of but little moment in other respects, it would seem that it should be chosen almost exclusively in reference to these two objects, namely:—1st. It should be chosen so as to disturb as little as possible existing notions and prejudices. 2nd. It should be chosen so as to render a sum of money, in the old notation, readily convertible into that of the new; and it is of the first importance that this should be effected without sacrificing even the smallest fraction of a farthing.

To accomplish these objects, the highest money-unit should be 250 pence, that is, 10*d.* more than our present pound or sovereign; the value of the next inferior unit would be 25*d.*: this *might* be called a *florin*. But I should certainly not recommend this denomination; there would be great commercial inconvenience in disturbing our nomenclature of accounts; whatever alterations in *value* be made, it is highly desirable that pounds, shillings, and pence should still remain, as at present, the only money-denominations in *accounts*.

With a view to the preservation of these terms, I would call the gold piece, value 250 pence, an IMPERIAL POUND; the silver piece, value 25 pence, an IMPERIAL SHILLING; and the silver piece, value 2½ pence, an IMPERIAL (or *silver*) PENNY; the present copper farthing and halfpenny, and the copper penny, remaining untouched. To distinguish between *imperial and common* or old money, the symbols for the former should be £. s. d.; those for the latter, *l.* s. d.

We thus have the following money-units expressed in *pence* (*d.*) :—

Imp. pound.	Imp. shilling.	Imp. penny.	Farthing.
250*d.*	25*d.*	2·5*d.*	·25*d.*

Although the decimal points appear in the last two denominations, yet in keeping accounts in imperial money the word *decimal* need never be implied, nor any reference to the notation of decimals be made, so that an uneducated person would have no more occasion to know that in casting up an account he was using the notation of decimals, than he need now know that in proceeding from pence to shillings he is using duodecimals. This will be seen more clearly presently.

To convert old money into imperial, we merely have to apply the principle that 1000 farthings = 1 imperial pound, thus: suppose the sum to be

	l.	*s.*	*d.*	
	9	13	7¼	
	20			
	193			
	12			
	2323			
	4			
			£.	s. d.
9*l.* 13*s.* 7¼*d.* =	£9·293	=	9	2 9·3

The amount in imperial money is therefore 9 pounds, 2 shillings and 9 pence 3 farthings. The point between the 9 and the 3 may be regarded as used, not for the purpose of marking the *decimal* of a penny, but for the convenience of separating the farthings from the pence.

It thus appears that for the conversion of old into imperial money, all we have to do is to reduce the proposed sum to *farthings*, and then to mark off *three* figures on the right for the shillings, pence, and farthings, the remaining figures on the left expressing the pounds.

If fractions of a farthing are included in the old notation, these same fractions (or their equivalent decimals) are to be appended in the new notation; thus, if the above sum had been 9*l.* 13*s.* 7¼*d.* + ¼*f.*, then the same sum in the new notation would have been £9. 2s. 9·3¼D., or £9·293¼, or £9·29325.

It is plain that in order to convert imperial money into old, we have only to unite the figures under £. S. D. into a single number, and to reduce that number of *farthings* into *l. s. d.* Or if the sum, imperial, be expressed wholly under £, then we are to remove the point three places to the right, and to convert the resulting farthings into *l. s. d.*

Thus, £9. 2s. 9.3¼D. = 9293¼ farthings.
Or, £9.29325 = 9293·25 „

The following specimen of a common account in the system here proposed, offers nothing to disturb existing notions, or to offend popular prejudice. The operations written on the right are those necessary to convert the total into old money :—

£.	s.	d.		
12	7	4·2		
8	9	2·4		4)32043 *farthings*.
3	6	7·5		
5	3	8·7		12)8010¾*d*.
0	5	·6		
0	8	·9		2,0)66,7*s*. 6*d*.
£32	0	4·3	*imperial* =	33*l*. 7*s*. 6¾*d*. *old*.

The total amount, *imperial*, is thirty-two pounds, 0 shillings, and fourpence three farthings. The amount in farthings, furnished at once by the row of figures, is 32043, which by reduction is 33*l*. 7*s*. 6¾*d*. old money.

I think that both in manuscript and in print, money-items should appear as in this specimen: a gap being left between pounds and shillings, and between shillings and pence, exactly as in the system in present use; and that for the sake of the uneducated, as well as for the general purposes of commerce, every form of expression borrowed from the arithmetic of decimals, ought to be avoided: the item £5. 3s. 8·7D. should be read 5 pounds 3 shillings and 8 pence 7 farthings—not 8 pence and 7 tenths.

In calculations connected with Interest, Annuities, Assurances, &c., mathematicians would no doubt work by decimals, in the ordinary form, as, indeed, they do at present, deeming it worth while to translate our inconvenient notation into the decimal notation: but in common accounts, and in all money statements, the term *decimal*—in consideration of the uneducated—should be suppressed, and all amounts stated in £. s. d.

The only coins, besides the recently-introduced florin, that need be recalled, if the system here proposed were to be adopted, are the three inferior silver coins; that is, the 3*d*., 4*d*., and 6*d*. pieces, to be replaced by the 2½*d*., the 5*d*., and the 10*d*. pieces. There would be no necessity abruptly to annihilate the present shilling: it might be suffered gradually to die out, as a thing not wanted, though not in the way. People would soon become reconciled to the *ten-piece*, which would speedily supplant the *twelve-piece*. Twenty-four of the former would make the old pound or sovereign, and twenty-five the imperial pound, which would thus represent a sovereign and a ten-piece.

The crown and half-crown might be retained—the former as a 30*d*. *piece, and the latter as a* 60*d*. piece; but they might be dispensed with,

though not without some inconvenience, at least, as respects the crown, unless replaced as hereafter suggested. Supposing, however, that they be both allowed to remain, the coins of commerce would then be as follows :—

GOLD.—Imperial pound (£.) ; half-pound.

SILVER.—Crown ; half-crown ; imperial shilling (s.) ; 10*d.* piece ; 5*d.* piece ; 2½*d.* piece, that is, the imperial or silver penny (D).

COPPER.—Penny (*d.*) ; halfpenny ; farthing, as at present.

The imperial shilling, or 25*d.* piece, would bear the same relation to the 10*d.* piece that the half-crown bears to the present shilling ; and the lowest three silver coins would have to one another the same relation as the three copper coins.

The silver coins here proposed are six in number : if the imperial shilling were made a little smaller in surface than the florin (which is, of course, supposed here to be withdrawn), and somewhat thicker, they could all six be easily distinguished, and any one of them could be identified, even in the dark.

If the half-crown were to be dispensed with—and it could be spared better than any of the other coins—the difference in the sizes would be abundantly palpable. But as soon as the public had become somewhat accustomed to the new coins here proposed, the half-crown and crown might advantageously give place to the 2s. and 4s. pieces. We should then have silver coins for 1D., 2D., and 4D., as also for 1s., 2s., and 4s. ; the gold half-pound being equivalent to 5s. It may be matter for consideration whether this scale should not be adopted at the outset.

The prefix *imperial* would no doubt go out of use with the disappearance of the old coins, till when it would effectually prevent confusion, and would also distinguish certain of the new coins by a dignifying appellation. The designations 10*d.* piece and 5*d.* piece, being significant of value—the copper penny being known,—would, perhaps, answer better than any merely arbitrary names. The tenpenny-piece and the fivepenny-piece were current in Ireland till the beginning of the present century.

It should be borne in mind that, although in conformity with the general practice in the recent discussions on the present topic, I have headed this paper "On the Decimal Coinage," the object really sought to be brought about by the proposed alteration in our monetary system, is the simplification of our *accounts*—the conducting our *monetary arithmetic* on an easier plan, so that time and labour may be saved, and, consequently, the risk of errors of operation diminished. As to the coins of commerce—the most convenient media for pecuniary transactions, *small* or *great*—these are matters for distinct, or, at least, for

supplementary consideration. There need not be a complete decimal *coinage;* this would be an inconvenience: the want of intermediate pieces of money would in many cases be seriously felt, especially by the poor. But our money of *accounts* should be purely and wholly on the decimal system. In *accounts*, no denomination should be mentioned but pounds, shillings, pence, and farthings, as at present; and for *these*, which, on the above plan, regularly descend in the decimal scale of value, there should unquestionably be appropriate representative *coins* bearing the same names;* the intervals of value between them should, however, be narrowed by the interposition of such other coins as would be most suitable for the wants of the community at large, in the usual pecuniary transactions of life. It seems to me that the copper halfpenny and penny, the silver 5*d.* piece, and 10*d.* piece, together with the existing half-crown and crown (preserving these for the present), would form coins of commerce, intermediate between the farthing, the silver or imperial penny (2½*d.*), the imperial shilling (25*d.*), and the gold half-pound, of sufficient range for all such transactions; and even from these, as before observed, the half-crown might be withdrawn.

I do not think there can be any objection to the two kinds of *penny* here proposed—the copper or common penny, and the silver or imperial penny,—that is not sufficiently overruled by the advantages attendant upon the admission of both. Our copper money is thus kept intact, both as to value and denomination, and the nomenclature, as also the symbols (£. S. D.) employed in accounts, are likewise retained without any violation of propriety.†

It may be as well to add that, in drawing up the present article, I have not had any of the numerous papers on the same subject before me: the germ of what is here given in much fuller detail, was communicated by me about a year ago in the pages of a popular periodical (see "The Family Tutor," for October, 1853); but it is very probable that, as well before that time as since, plans may have been proposed differing but little from the project here recommended. But such of these as have come under my own notice seem to enforce an exclusively decimal *coinage* as much as a decimal system of accounts; and to imply that the former is a necessary consequence of, or at least a needful accompaniment to, the latter; and thus *dimes*, and *decimes*, and *minims*,

* Such, however, is not the case at present, at least as to *name;* the *pound* of accounts is the *sovereign* of commerce. This double nomenclature, which answers no purpose but to confuse, should be abolished.

† It is no uncommon thing, in other countries, for coins of different metal and value to bear a common name. In Sweden, there are the copper dollar, the silver dollar, and the rix dollar; of which the second is thrice the value of the first, and *the third thrice the* value of the second.

If the plan proposed above were to be adopted, the denomination silver *penny would be retained* after the distinction imperial had become unnecessary.

and *mites*, and other terms of equally insignificant import, have been proposed to mark certain of these decimal divisions in our coins.

Such extensive, and, moreover, somewhat repulsive innovations, are not at all required by the exigencies of the case: our copper coinage does very well as it is,—much better than any substitute for it would do that should *approximate only* to the present current values, by disregarding fractions of a farthing, however minute the discrepancy may be. A system affected with *any error at all* of this kind, could not fail to produce dissatisfaction among the poor; and it would, moreover, be a national discredit.

The designation *imperial*, that I have proposed to give to the coins of account, was of course suggested to me by the like epithet in the changed system of weights and measures, which came into operation in 1826: I am only proposing to add imperial money to our imperial weights and imperial measures. The public are already familiar with the term, which—unlike the terms above noticed—is euphonious and dignified; and it is merely introduced as a temporary prefix, necessary only during the transition state—a period of some little time—from the old system to the new.

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